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ABSTRACT

This programed instruction study guide is one of a series that form a first-year algebra course. Structured in a multiple-choice question-answer format with scrambled pages, it is intended to be used in conjunction with a computer-managed instructional system. The following topics are covered in Volume 15: solving digit, motion, and age problems; solving problems involving fractions, rational numbers, determining roots of numbers, and irrational numbers. Reading and homework assignments are taken from the text "Modern Algebra - Book I" by Dolciani. (Related documents are SE 015 854 - SE 015 869.) (DT)

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PROGRAMMED MATH CONTINUUM

level one

ALGEBRA

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VOLUME

15

NEW YORK INSTITUTE OF TECHNOLOGY
OLD WESTBURY, NEW YORK

ED 075216

P R O G R A M M E D M A T H C O N T I N U U M

LEVEL ONE

A L G E B R A

VOLUME 15

New York Institute of Technology

Old Westbury - New York

PREFACE

This volume is one of a set of 18

that form a complete course

in

ALGEBRA - LEVEL ONE

The volume has been structured

in a multiple choice question-answer format,

with the pagination scrambled

and

is to be used in conjunction with

a program control console

utilizing

punch card input.

It is one exhibit in the demonstration of a model

developed under the direction of

the U.S. Department of Health Education and Welfare

Project 8-0157

at the

New York Institute of Technology

Westbury, New York

VOLUME 15

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1	1	$\frac{1}{1}$
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1	4	$\frac{133}{1}$
1	5	$\frac{176}{1}$

VOLUME 15

This volume covers the following material as shown in this excerpt from the Syllabus:

SEGMENT	DESCRIPTION	REFERENCE BOOK SECTION		
		DOLCIANI	DRESSLER	DODES
1	Solution of digit problems	10-7	16-4	
2	Solution of motion problems	10-8	16-4	7-1
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READING ASSIGNMENT

VOLUME 15

Before you begin to answer the questions in this STUDY GUIDE you should read the pages indicated.

SEGMENT	FROM PAGE	TO PAGE
1	381	382
2	383	384
3	385	386
4	397	403
5	403 407	405 409

Modern Algebra Book I
Dolciani, Berman and Freilich
Houghton Mifflin, 1965

Read EVERYTHING contained in these pages.

EXAMINE every illustrative problem

Write in your NOTEBOOK:

1) Every RULE that has been stated
2) Every DEFINITION that has been presented
3) Solve at least ONE PROBLEM of each type covered in the lesson.

If you wish additional information for enrichment purposes consult:

Algebra I
Dodes and Greitzer
Hayden Book Co., 1967

You will be given additional notes at various places in the STUDY GUIDE. These, too, should be entered in your NOTEBOOK.

HEW
PMC

E

HOMEWORK ASSIGNMENT

VOLUME NO. 15

BOOK: DOLCIANI

HOMEWORK QUESTION NO.	PAGE NO.	EXAMPLE NUMBER	MBO REFERENCE
1	382	1	15110
2	382	3	15110
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16	403	14, 15, 16	15450
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19	410	Written Ex: 11, 12, 17	15523
20	410	Written Ex: 13	15522

GENERAL INSTRUCTIONS

Ask your teacher for:

PUNCH CARD
PROGRAM CONTROL
ANSWER MATRIX

When you are ready at the PROGRAM CONTROL

Insert the PUNCH CARD in the holder
Turn to the first page of the STUDY GUIDE
Read all of the instructions
Read the First Question

Copy the question
Do your work in your notebook
Do all of the computation necessary
Read all of the answer choices given

Choose the Correct answer
(remember, once you've punched the card
it can't be changed)

Punch the card with the STYLUS

Read the instruction on the PROGRAM CONTROL
(it tells you which page to turn to)

TURN TO THAT PAGE:

If your choice is <u>not correct</u> you will be given additional hints, and will be directed to return to the question and to choose another answer.

If your choice is <u>correct</u> then you will be directed to proceed to the next question located immediately below, on the same page.

If you have no questions to ask your teacher now, you can turn the page and begin. If you have already completed a SEGMENT turn to the beginning of the following segment;

CHECK THE PAGE NUMBER BY LOOKING AT THE TABLE OF CONTENTS

Volume 15 Segment 1 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS 48 and 50 2 6 (Sequence Number)

54 and 56 0 4 (Type of Punch Card)

60 and 62 1 5 (Volume Number)

66 and 68 0 1 (Segment Number)

Your READING ASSIGNMENT for this Segment is pages 381 - 382 .

INTRODUCTORY NOTE

We enjoy solving problems for the same reason that we enjoy doing a cross-word puzzle. Solving a puzzle gives one a feeling of accomplishment, and no other reward is needed for engaging in this activity. However, the type of problem that we will study now has a value beyond the mere fun of solving a puzzle. The problems of this segment will give you a greater insight into the nature of numbers, especially the decimal system of writing numbers. You will discover in your future work in mathematics that an understanding of numbers is basic to almost all work in mathematics. One of the greatest mathematicians who ever lived, Freidrick Gauss, said: "Mathematics is the Queen of the sciences, and numbers is the Queen of Mathematics."

You will now be asked a series of questions to draw your attention to the more important points.

Question 1

If the tens digit of a two-digit number is x and the units digit is y, which do you recognize as the number?

(A) $x + y$

(B) $10x + y$

(C) $10y + x$

(D) $10t + u$

$\frac{2}{1}$

Let us clarify the meaning of the word digits. Consider any number less than 100 . For example, the number 68 . This number is composed of the two digits 6 and 8 . In any whole number less than 100 and more than 9 the first digit reading from left to right is called the tens digit. This digit can be any integer number from 1 to 9 . The second number is called the units digit. This digit can be any integer number from 0 to 9 .

Now, since the tens digit cannot be a number greater than 9 your choice is incorrect.

Please return to page $\frac{30}{2}$ and try question 3 again.

$\frac{2}{2}$

We do not agree. The units digit of a two digit number can have any value from 0 to 9 while the tens digit of a two digit number can have values ranging from 1 to 9 . Write some two digit numbers down on a separate piece of paper to convince yourself of this fact.

Please return to page $\frac{26}{2}$ and try question 8 again.

3/1

Please keep in mind that the letter t denotes the tens digit, and the letter u denotes the units digit. When we are talking about digits, only the letters t and u are involved. Only when we are talking about the number does the coefficient 10 appear. Read the question again, and you will note that the word number does not appear in this problem.

Please return to page 22/2 and try question 5 again.

3/2

The only time a two digit number remains the same upon reversing its digits is when both digits are alike. Thus, for example, the number 77 remains 77 when its digits are reversed.

Please return to page 14/2 and try question 11 again.

$\frac{4}{1}$

The largest numeral that appears in the decimal system is (9) , and 10 times the whole number p is larger than 9 . Therefore, we cannot agree with your choice.

The number (10p + q) is composed of two digits p and q .
The tens digit is p and the units digit is q . The value of the tens digit is 10p and the value of the units digit is q .

Please return to page $\frac{20}{2}$ and try question 2 again.

$\frac{4}{1}$

If a two digit number is represented by (10m + n) , then m and n are the digits and m + n is the sum of these digits. By the Commutative Property n + m is also the sum of the digits. Thus, for example, the number 72 means 70 + 2 and the sum of its digits is 7 + 2 or 2 + 7 .
You incorrectly chose the sum of the digits for the original number which was obtained by reversion of the digits of the number given. The number obtained from 72 when the digits are reversed is 27 not 2 + 7 .

Please return to page $\frac{32}{2}$ and try question 7 again.

$\frac{5}{1}$

If the number 47 is written with its digits reversed, what would this number be? That's easy, you say, it would be 74 . Now, read the question again. This time, answer it correctly.

Please return to page $\frac{16}{2}$ and try question 4 again.

$\frac{5}{2}$

Consider the following two digit numbers:

20 , 50 , 70

53 , 83 , 23

16 , 76 , 96

Have you changed your mind about the choice you made?

Please return to page $\frac{21}{2}$ and try question 10 again.

$\frac{6}{1}$

The place value of the tens digit is not the same as its face value. Thus, for example, if the tens digit of a two digit number is 7 and the units digit is 5 , the number is 70 + 5 = 75 and not 7 + 5 = 12 .

Please return to page $\frac{1}{2}$ and try question 1 again.

$\frac{6}{2}$

We do not agree. The equation you chose states that the two digit number,

$$10w + r = 8$$

Since the smallest possible 2 digit number in the decimal system is 10 , a 2 digit number equal to 8 isn't possible.
You must distinguish between the number $10t + u$ and the sum of the digits, $t + u$.

Please return to page $\frac{17}{2}$ and try question 6 again.

A number such as 05 is not considered a two digit number, but rather a one digit number.

In a two digit number, the tens digit cannot be zero. Consider the number

$$75 = 70 + 5$$

The place value of the tens digit of this number is 70 .

Please return to page 30/2 and try question 3 again.

The equation that you chose states that a two digit number is equal to 5 . You know that the smallest two digit number is 10 . Thus, your choice cannot be right. Keep in mind that the number is $10t + u$. The sum of its digits is simply the sum of the letters used to denote these digits. Can ycu now make the correct choice?

Please return to page 33/2 and try question 13 again.

$\frac{8}{1}$

You don't mean what you said. The question states "the units digit is twice the tens digit." Take another look at what you selected.

Please return to page $\frac{22}{2}$ and try question 5 again.

$\frac{8}{2}$

Consider any two digit number where the tens digit is the larger of the digits. For example, think of the number 82 . What happens to the size of this number when its digits are reversed?

Please return to page $\frac{14}{2}$ and try question 11 again.

Yes, we agree.

The reversed two digit number is 65 . Therefore, this number before its digits were reversed was 56 . Hence, 5 was the tens digit and 6 was the units digit.

Please proceed to question 9 which follows.

Question 9

If t represents the tens digit of a two-digit number, N , where $N < 93$

Choose the value which t cannot have.

(A) 0

(B) 3

(C) 6

(D) 9

$\frac{10}{1}$

We do not agree. A number represented by the expression $10y + x$ means that the tens digit of this number is y. Read the question over again, and you will note that this is not the information that is given.

Please return to page $\frac{1}{2}$ and try question 1 again.

$\frac{10}{2}$

The expression $(10r + w)$ is the <u>number</u> $(10w + r)$ with its digits reversed. Note that the word number is underlined. Why? When the word <u>number</u> is mentioned, you should immediately know that the tens digit has the coefficient 10 when you represent it. Weren't you asked for a representation of the sum of the digits?

Please return to page $\frac{17}{2}$ and try question 6 again.

$\frac{11}{1}$

Have you read the question carefully and considered the other choices offered? One of them is correct. You were not asked for the place value of the digit. Based on its position, you were asked for the digit itself.

Please return to page $\frac{30}{2}$ and try question 3 again.

$\frac{11}{2}$

Read the last four words of the question again. It states that "the number is 12 ." Your choice states that the sum of its digits is 12 .

Please return to page $\frac{23}{2}$ and try question 12 again.

$\frac{12}{1}$

We do not agree. You should take a critical look at every algebraic statement you write to see "if it makes sense."

The equation that you chose,

$$u = 2(10t)$$

$$u = 20t$$

states that the units digit is 20 times the tens digit.

Please return to page $\frac{22}{2}$ and try question 5 again.

$\frac{12}{2}$

Take a look at the equation that you chose. It states that the number increased by 5 is equal to the sum of its digits. Now, read the question. Do you see that this is not what the problem calls for?

Please return to page $\frac{33}{2}$ and try question 13 again.

$\frac{13}{1}$

In writing a number of two digits in the form $10p + q$ the digit which has a coefficient of 1 is the units digit. Thus, in this question q is the units digit.

Please return to page $\frac{20}{2}$ and try question 2 again.

$\frac{13}{2}$

The given two digit number is $10m + n$. Your choice is $n + 10m$. If you recall the commutative property of addition, you will note that

$$10m + n = n + 10m$$

Thus, you did not answer the question.

Please return to page $\frac{32}{2}$ and try question 7 again.

14
1

Your choice is correct. The units digit of a number in the decimal scale cannot be larger than 9 . It must be a single digit.

Please proceed to question 11 which follows.

14
2

Question 11

In a certain number of two digits, the tens digit is larger than the units digit. If the digits are reversed, apply your knowledge of algebra to determine which statement is true.

(A) The number remains the same.

(B) The number becomes larger.

(C) The sum of the digits remains the same.

(D) The sum of the digits becomes smaller.

$\frac{15}{1}$

Yes, but in this problem you were asked to work with the letters x and y , and not with the letters t and u . Please reconsider your choice.

Please return to page $\frac{1}{2}$ and try question 1 again.

$\frac{15}{2}$

We do not agree. Please write some two digit numbers on a separate piece of paper. For example, write the numbers,

37 , 65 , 94

How do you feel about your choice now?

Please return to page $\frac{9}{2}$ and try question 9 again.

$\frac{16}{1}$

Very good. You made the correct choice. The place value of the tens digit of a two digit number is 10 times its face value. Thus, a place value of 20 indicates that the face value is 2 .

Please proceed to question 4 which follows.

$\frac{16}{2}$

Question 4

If the number 47 is written with the digits reversed, choose the place value of the resulting units digit.

(A) 7

(B) 4

(C) 70

(D) 40

Very good. Your choice is correct.

The digits of a two digit number are represented by t and u where t denotes the tens digit and u denotes the units digit. The equation $u = 2t$ states that u, the units digit is equal to twice t the tens digit.

This is the relation expressed by the question.

Please proceed to question 6 which follows.

Question 6

If r represents the units digit and w the tens digit, choose the equation which states that the sum of the digits is 8.

(A) $10w + r = 8$

(B) $r + w = 8$

(C) $10r + w = 8$

(D) None of these.

$\frac{18}{1}$

Always check the meaning of an algebraic statement that you write.

Your choice states that the product of the digits is 12 .

Please read the question once more.

Please return to page $\frac{23}{2}$ and try question 12 again.

$\frac{18}{2}$

Let us again read the part of the question that needs to be translated into an algebraic equation. "The units digit exceeds one-half the tens digit by 3 ." What is the units digit? What is the tens digit? Surely, you can see your error. Is '10t' the tens digit?

Please return to page $\frac{38}{2}$ and try question 14 again.

$\frac{19}{1}$

We do not agree. One of the letters does have the correct answer next to it. Read the problem carefully and make the necessary substitutions.

Please return to page $\frac{32}{2}$ and try question 7 again.

$\frac{19}{2}$

We do not agree. Let us write a few numbers and alongside each number we write the number with its digits reversed. Thus,

(37 , 73)

(58 , 85)

(62 , 26)

Now find the sum of the digits of each pair of 2 digit numbers in the parentheses. Have you arrived at a conclusion?

Please return to page $\frac{14}{2}$ and try question 11 again.

$\frac{20}{1}$

Yes, your choice is correct.

If the tens digit of a two digit number is denoted by x, then its place value is $10x$. Furthermore, if y denotes the units digit of this number, the number is represented by the expression $10x + y$.

Please proceed to question 2 which follows.

$\frac{20}{2}$

Question 2

If a two-digit number is represented as $10p + q$ which do you recognize as the tens digit?

(A) p

(B) 10p

(C) q

(D) 10q

You made the correct choice.

The tens digit of a two digit number can have values ranging from 1 to 9 . It cannot have the value 0 . For example, 07 is not a two digit number.

Please proceed to question 10 which follows.

Question 10

If u represents the units digit of a two-digit number N where $9 < N < 79$ the value which u cannot have.

(A) 0

(B) 3

(C) 6

(D) 10

$\frac{22}{1}$

Yes, your choice is correct.

The number 47 with its digits reversed is 74 . The units digit is now 4 . The place value and the face value of the units digit is always the same.

It is the units digit.

Please proceed to question 5 which follows.

$\frac{22}{2}$

Question 5

Using t to represent the tens digit and u the units digit, choose the equation which states that the units digit is twice the tens digit.

(A) $10t = 2u$

(B) $t = 2u$

(C) $u = 2(10t)$

(D) $u = 2t$

Very good. You made the correct choice. If we let t denote the tens digit and u denote the units digit, then the sum of these digits is $t + u$. When the digits of the number are reversed, the sum of the digits of the new number is $u + t$. By the commutative property of addition,

$$t + u = u + t$$

Therefore, the sum of the digits remains unchanged when the digits of a number are reversed.

Please proceed to question 12 which follows.

Question 12

Using t for the tens digit and u for the units digit, choose the equation which states that the number is 12.

(A) $t + u = 12$

(B) $tu = 12$

(C) $10t + u = 12$

(D) $10u + t = 12$

$\frac{24}{1}$

We do not agree. One of the letters does have the correct answer next to it. Please reconsider your choice. Translate directly from English to Algebra.

Please return to page $\frac{17}{2}$ and try question 6 again.

$\frac{24}{2}$

Let us see what your choice says. You chose the equation,

$$u + 3 = \frac{1}{2}(10t)$$

$$u + 3 = 5t$$

Your choice states that the units digit increased by 3 is equal to 5 times the tens digit. Clearly, this is not what is being asked. Keep in mind that the digits are simply t and u .

Please return to page $\frac{38}{2}$ and try question 14 again.

We do not agree with the number designated by $10p + q$. The expression $10q$ would mean ten times the units digit q. This is not what we are looking for.

Please return to page 20/2 and try question 2 again.

We will help you get started. Let t denote the tens digit and u denote the units digit of the two digit number. Then, $10t + u$ represents the original number and $10u + t$ represents this number with its digits reversed.

Now read the problem once more. You should be able to write two equations with t and u as the unknowns. Since you know how to solve two equations in two variables, we will say no more, excep' don't forget to check your answers.

Please return to page 48/2 and try question 16 again.

$\frac{26}{1}$

Yes, you made the correct choice. If a two digit number is denoted by

$$10m + n$$

this number with its digits reversed is denoted by

$$10n + m$$

When the digits of a two digit number are reversed, the tens digit becomes the units digit, and the units digit becomes the tens digit. Thus, for example,

$$46 = 40 + 6$$

and

$$64 = 60 + 4$$

Please proceed to question 8 which follows.

$\frac{26}{2}$

Question 8

After reversing the order of the digits, the face value of the new tens digit is 6 and the face value of the new units digit is 5 . Choose the face values of the original digits.

(A) Tens digit was 5 , units digit was 60 .

(B) Tens digit was 50 , units digit was 6 .

(C) Tens digit was 60 , units digit was 5 .

(D) Tens digit was 5 , units digit was 6 .

We do not agree. If the letter t denotes the tens digit and the letter u denotes the units digit of a two digit number, the coefficient 10 is written immediately before the letter t in the representation of the number.

Actually, you chose the number whose digits are the reverse of the number in question.

Please return to page $\frac{23}{2}$ and try question 12 again.

We believe that you really worked out this problem carefully. But, did you check your answer? No. The problem states that the tens digit is 1 more than the units digit. Now, in the number 56 , the tens digit is 1 less than the units digit. Please go over your work, and this time check your answer with the words of the problem.

Please return to page $\frac{36}{2}$ and try question 17 again.

$\frac{28}{1}$

We agree that the number is represented by the expression $10t + u$ and that the sum of its digits is represented by $(t + u)$. The equation you chose states that the number is equal to the sum of its digits decreased by 5.

Please be more careful.

Please return to page $\frac{33}{2}$ and try question 13 again.

$\frac{28}{2}$

Have you set up the variables and the relationships carefully?

Denoting the tens digit by t and the units digit by u the problem can be translated into algebra and gives the equations

$$t = 2u + 1 \text{ and}$$

$$t - u = 3$$

Please continue from here. Solve the above equations and check your answers.

Please return to page $\frac{35}{2}$ and try question 19 again.

A correct translation into algebra of the statement of your choice is the equation

$$(10t + u) + 1 = 2u$$

This equation is not the same as the one given.

Please return to page 43/2 and try question 15 again.

Don't forget to list the variables and to translate the relationships into the two equations. If you use t to represent the tens digit, and u to represent the units digit, you will get the equations

$$(1) \quad t + u = 6$$

and

$$(2) \quad t = 2u$$

Solving these equations will give the values of t and u. After you find these values, you will discover that your choice is incorrect.

Please return to page 46/2 and try question 18 again.

$\frac{30}{1}$

Yes, your choice is correct. If a two digit number is represented by $10p + q$ then the tens digit of this number is p. Similarly, if a two digit number is represented by $10m + n$ the tens digit is m. In general, you recognize the tens digit of a two digit number in the form $10t + u$ by the letter that has 10 for its coefficient.

Please proceed to question 3 which follows.

$\frac{30}{2}$

Question 3

If the place value of the tens digit in a number is twenty, which do you recognize as the tens digit?

(A) 20

(B) 0

(C) 2

(D) None of these.

$\frac{31}{1}$

Almost, but not quite.

Don't pat yourself on the back. Almost, in mathematics still means wrong.

The equation you chose; namely,

$$u + 3 = \frac{1}{2}t$$

states that the units digit increased by 3 equals one-half the tens digit. The question on the other hand, states that the units digit is more than one-half the tens digit. Therefore, if 3 is added to the larger digit it cannot be said to be equal to the other quantity.

Please return to page $\frac{38}{2}$ and try question 14 again.

$\frac{31}{2}$

Before you choose an answer you should check it.

Your answer does not check with the second condition of the problem. The number 65 does not equal to 1 more than 6 times the sum of its digits.

That is

$$65 \neq 6(6 + 5) + 1$$

Please return to page $\frac{36}{2}$ and try question 17 again.

$\frac{32}{1}$

Very good. Your choice is correct. If we let

r = the units digit,

w = the tens digit, then

r + w represents the sum of these digits.

Thus, r + w = 8 is the correct equation.

The sum of the digits of the number represented by (10t + u) is (t + u).

Please proceed to question 7 which follows.

$\frac{32}{2}$

Question 7

The number formed by reversing the digits of a two-digit number is 10m + n .

Choose the correct representation of the original number.

(A) n + m

(B) n + 10m

(C) 10n + m

(D) None of these.

Yes, we agree with your choice.

If t denotes the tens digit and u denotes the units digit, the number is denoted by $10t + u$. Since we are told that the number is 12, the correct equation is

$$10t + u = 12$$

Please proceed to question 13 which follows.

Question 13

If t represents the tens digit and u represents the units digit of a two-digit number, apply your knowledge to find the equation which states that the number is 5 more than the sum of its digits.

(A) $10t + u = t + u + 5$

(B) $10t + u = 5$

(C) $10t + u + 5 = t + u$

(D) $10t + u = t + u - 5$

$\frac{34}{1}$

The correct translation of the statement of your choice into algebra is

$$10t + u = 2(u + 1)$$

Clearing parentheses, we get

$$10t + u = 2u + 2$$

Notice that this is in contradiction to the equation given.

Please return to page $\frac{43}{2}$ and try question 15 again.

$\frac{34}{2}$

We do not agree. Follow along with us as we do a similar problem together. The sum of the digits of a 2 digit number is 6 . If the digits are reversed, the new number will be 6 more than 3 times the original number. Find the original number.

Solution: Let t = the tens digit

u = the units digit, then

$10t + u$ = the number, and

$10u + t$ = the number with the digits reversed.

Now, the first sentence tells us that

"the sum of the digits is 6 "

$$t + u = 6$$

The second sentence of the problem tells us that

"the reverse number" is 3 times the number plus 6 .

$$10u + t = 3(10t + u) + 6$$

Simplifying the latter equation, we get

$$10u + t = 30t + 3u + 6 \quad \text{<- } 3u, - t$$

$$7u = 29t + 6$$

Substituting the value of u from the first equation, we have

$$7(6 - t) = 29t + 6$$

$$42 - 7t = 29t + 6 \quad \text{<+ } 7t - 6$$

$$36 = 36t$$

$$1 = t \text{ and } u = 6 - 1$$

$$= 5$$

Hence, the original number is 15 .

Check: The sum of its digits is $1 + 5 = 6$. The number with the digits reversed is 51 and $51 = 3(15) + 6 = 45 + 6 = 51$

Please return to page $\frac{42}{2}$ and try question 20 again.

Very good. You made the correct choice. Let's review this problem together.

I Let t = the tens digit,
u = the units digit.

II The first sentence translated into algebra yields the equation,

"the sum of the digits is 6"

$t + u = 6$ III Check: $(4 + 2 = 6)$

The second sentence of the question states that

"the tens digit is twice the units digit:

$t = 2u$ Check: $(4 = 2 \times 2)$

Substituting this value of t in the first equation, we get

$$2u + u = 6$$
$$3u = 6$$
$$u = 2 \text{ and } t = 2u$$
$$= 4$$

Therefore, the number is 42 . We check the values by showing the relationships are satisfied by our choice.

Please proceed to question 19 which follows.

Question 19

The tens digit of a two-digit number exceeds twice the units digit by 1 . The difference of the digits is 3 . Apply your knowledge to find the correct statement.

(A) the tens digit plus twice the units digit is 12 .

(B) the units digit plus twice the tens digit is 9 .

(C) the tens digit plus twice the units digit is 9 .

(D) the tens digit plus twice the units digit is 5 .

$\frac{36}{1}$

Very good. Your choice is correct. You could have solved this problem as follows:

				CHECK
I	Let	t =	the tens digit,	= 3
		u =	the units digit,	= 6
		$10t + u$ =	the original number,	= 36
		$10u + t$ =	the number with the digits reversed.	= 63

II The first sentence of the question tells us that

(1) $t = \frac{1}{2} u$

The second sentence yields the equation,

$(10t + u) + (10u + t) = 99$ [Collect

$11t + 11u = 99$ ∝ ÷ 11

(2) $t + u = 9$

Substituting equation (1) $t = \frac{1}{2} u$ in this equation, we get,

$(\frac{1}{2}u) + u = 9$ [Collect

$\frac{3}{2}u = 9$ ∝ (2)

$3u = 18$ ∝ ÷ 3

$u = 6$

and $t = \frac{1}{2}u$

$t = \frac{1}{2}(6)$

$t = 3$

Hence, the number is 36 and the sum of its digits is $3 + 6 = 9$.
The sum of the number + the reverse $= 99$

III Furthermore, $36 + 63 = 99$. The answer checks.

Please proceed to question 17 which follows.

$\frac{36}{2}$

Question 17

The tens digit of a two-digit number is one more than its units digit. If the number is one more than 6 times the sum of its digits, choose the correct number.

(A) 56 (B) 65 (C) 43 (D) 34

$\frac{37}{1}$

Not quite. Distance is measured in units such as inches, feet, yards, and the like. Do you see your error?

Please return to page $\frac{49}{2}$ and try question 1 again.

.....

$\frac{37}{2}$

Perhaps you are not sure of the meaning of the word "downstream" in problems of this sort. When a boat is sailing downstream, it is moving in the same direction as the current of the water. Thus, the current is helping the boat along.

You then see the relationship between the two motions since they are both in the same direction. The resulting rate is the sum of the two individual rates.

Please return to page $\frac{47}{2}$ and try question 4 again.

$\frac{38}{1}$

Very good. You made the correct choice. If t represents the tens digit of a two digit number and u represents the units digit, then the number is denoted by $10t + u$. Hence, the statement that the number is 5 more than the sum of its digits is

$$10t + u = t + u + 5$$

Please proceed to question 14 which follows.

$\frac{38}{2}$

Question 14

If t represents the tens digit and u represents the units digit of a two-digit number, apply your knowledge to find the equation which states that the units digit exceeds half the tens digit by 3.

(A) $u = \frac{1}{2}(10t) + 3$

(B) $u + 3 = \frac{1}{2}(10t)$

(C) $u + 3 = \frac{1}{2}t$

(D) None of these.

$\frac{39}{1}$

The correct translation into algebra of the equation of your choice is

$$2u = 10t + u + 1$$

The above equation can be written as

$$2u - 1 = 10t + u$$

Compare this with the given equation?
Do you see your mistake?

Please return to page $\frac{43}{2}$ and try question 15 again.

.....

$\frac{39}{2}$

You seem to be having some trouble in determining the relationships between the parts of the problem.
If the problem stated that a boy swims 3 miles in 6 hours, you would say that he swims $\frac{1}{2}$ mile in each hour.

Now if you analyze the values given in this example, you would see that:

Distance = Rate x Time

3 miles = Rate x 6 hours $\quad \div 6$ hours

$$\frac{3 \text{ miles}}{6 \text{ hours}} = \text{Rate}$$

$$\frac{3 \text{ miles}}{6 \text{ hours}} = \frac{1 \text{ mile}}{2 \text{ hours}}$$

or $\quad (\frac{1}{2}) \frac{\text{mile}}{\text{hour}}$

Keep in mind the meaning of rate. Rate means the distance covered in a unit of time. In this question, rate means the distance covered in 1 hour.

Please return to page $\frac{52}{2}$ and try question 3 again.

$\frac{40}{1}$

We do not agree. The basic formula relating Distance, Rate, and Time is

$$\text{Distance} = \text{Rate} \times \text{Time}$$

A formula is an equation. You can, therefore, use the methods of solving equations to transform the basic formula into its other forms.

$$D = RT$$

Now, what operation must be performed on both sides of the equation to solve for T ?

Please return to page $\frac{59}{2}$ and try question 2 again.

.....

$\frac{40}{2}$

Examine your choice $(300 - 2w)$.
It is a number less than 300 . Now, if a plane can fly into the face of a wind at a certain rate of speed, it certainly can go a good deal faster with the wind at its tail.

Please return to page $\frac{66}{2}$ and try question 6 again.

We do not agree.

You should have noticed that your answer is wrong. In the first place, the tens digit should be one more, instead of one less, than the units digit. In the second place, 6 times the sum of the digits of the number 34 is

$$6(3 + 4) = 42$$

One more than this number is 43 . But

$$43 \neq 34$$

Hence, your choice does not check with either of the conditions of the problem. Don't get upset; start all over again as follows:

Let t = the tens digit,

u = the units digit; and

$10t + u$ = the number.

Now, the first statement gives you the equation

$$t = u + 1$$

Read the second sentence of the question and write one more equation. Solve the two equations simultaneously and <u>check</u> your answers.

Please return to page 36/2 and try question 17 again.

No, your answer is not correct, but it does bring up an interesting situation. If the stream travels at the rate of 3 mph and the boat travels at a rate <u>less</u> than that in still water, say 2 mph , then if the boat were heading <u>upstream</u> the net effect would be 3 - 2 or 1 mph <u>downstream</u>.

However, whatever the rate of this boat, the assumption made is that it, too, is <u>heading</u> downstream with the current. Now even in this case where the speed of the boat is less than that of the stream, the answer would be the <u>sum</u> of the two rates.

Now return to page 47/2 and choose the correct answer.

$\frac{42}{1}$

Very good. You made the correct choice. We start by letting

I t = the tens digit,

u = the units digit.

II We are told that the tens digit exceeds twice the units digit by 1 . Writing this sentence as an algebraic equation, we obtain

$t = 2u + 1$ Check: $(5) = 2(2) + 1$

Furthermore, the difference of the digits is 3 . Since $t - u$ stands for the "difference" we have

$t - u = 3$ $(5) - (2) = 3$

Substituting the expression for t from the first equation, we get

$$(2u + 1) - u = 3 \qquad t = 2(2) + 1$$
$$u + 1 = 3 \qquad = 5$$
$$u = 2$$

The number is, therefore, 52 .

Check: $5 = 2(2) + 1$

$= 5$

and $5 - 2 = 3$

Please proceed to question 20 which follows.

$\frac{42}{2}$

Question 20

The tens digit of a two-digit number is 2 less than the units digit. The number with the digits reversed exceeds 6 times the sum of the digits by 4 . Apply your knowledge to find the correct statement about the value of t , the tens digit; and u , the units digit.

(A) $t + u < 10$ (B) $tu = 24$

(C) t and u are both odd (D) t is even, but u is odd.

. We agree with your choice. None of the other choices were correct.

The correct translation of the problem into algebra is

$$u = \frac{1}{2}t + 3$$

This equation states that the units digit equals one-half the tens digit plus 3 . That is what is meant when we say that the units digit exceeds the tens digit by 3 .

Please proceed to question 15 which follows.

Question 15

If t represents the tens digit and u the units digit, apply your knowledge to find the statement equivalent to the equation

$$10t + u = 2u + 1$$

(A) The number increased by 1 equals twice the units digit.

(B) The number is twice one more than the units digit.

(C) The number is one more than twice the units digit.

(D) Twice the units digit is one more than the number.

$\frac{44}{1}$

We do not agree. Sometimes it helps to convert an algebraic problem into a similar numerical problem. Suppose we replace the letter m by 250 and the letter h by 3 . The question would then read as follows: if an airplane flies at the rate of 250 miles per hour for 3 hours, what is the distance it will cover? Yes, you say 750 miles. How did you get this answer?

Please return to page $\frac{49}{2}$ and try question 1 again.

.....

$\frac{44}{2}$

You may have misunderstood the question. In spite of the current working <u>against</u> the boat, the boat is able to maintain a speed of 5 miles per hour.

Please return to page $\frac{60}{2}$ and try question 5 again.

Nice Work. You made the correct choice. We will solve this problem with you.

			Answer
Let	t	= the tens digit,	= 4
	u	= the units digit,	= 6
	$10t + u$	= the original number, and	= 46
	$10u + t$	= the number with the digits reversed.	= 64

We are told that the tens digit is 2 less than the units digit. Algebraically, we say

$t = u - 2$ Check: $4 = 6 - 2$

Furthermore, we are told that the number with the digits reversed exceeds 6 times the sum of the digits by 4 . Hence,

$64 = 6(6 + 4) + 4$

$10u + t = 6(t + u) + 4$ [D

Simplifying this equation, we get

$10u + t = 6t + 6u + 4$ ⟨-6u, -1t

$4u = 5t + 4$

Replacing t by $(u - 2)$ we get

$4u = 5(u - 2) + 4$ [D

$4u = 5u - 10 + 4$ ⟨-5u

$-u = -6$ ⟨(-1) and $t = 6 - 2$

$u = 6$ $t = 4$

Thus, the number is 46 and the product of the digits 4 and 6 is 24 .

You have now completed this Segment.

Hand in your PUNCH CARD. You should have entered in your NOTEBOOK the following definitions and formulas:

(1) In dealing with digit problems, let t = the tens digit,
u = the units digit, then
$10t + u$ = the number
$10u + t$ = the number with the digits reversed, and
$t + u$ = the sum of the digits.

You should now do problems 1, 2, 3, and 4 of your HOMEWORK ASSIGNMENT.

46
1

Very good. You made the correct choice. You could have proceeded as follows:

				Answer
I	Let	t	= the tens digit	= 4
		u	= the units digit	= 3
		$10t + u$	= the number	= 43

II The first statement of the question tells us that

(1) $t = u + 1$ Check: $4 = 3 + 1$

The second statement of the question tells us that

$10t + u = 6(t + u) + 1$ [D Check: $43 = 6(4 + 3) + 1$, $43 = 6(7) + 1$

Simplifying this equation, we get

$10t + u = 6t + 6u + 1$ ≮ $-6t, -u$

(2) $4t = 5u + 1$

Substituting the value of t from the first equation, we obtain

$4(u + 1) = 5u + 1$ [D

$4u + 4 = 5u + 1$ ≮ $-5u, -4$

$-u = -3$ ≮ ÷ (-1)

$u = 3$ and $t = u + 1$

$= 3 + 1$

$= 4$

Hence, the number is 43 .

The check can be arranged alongside of the list of variables and the relationship.

Please proceed to question 18 which follows.

46
2

Question 18

The sum of the digits of a two-digit number is 6 . If the tens digit is twice the units digit, apply your knowledge to find the correct choice for the value of the number N .

(A) $N > 40$ (B) $N < 20$ (C) $20 < N < 30$ (D) $30 < N < 40$

We agree with your choice.

To answer the question, we use the relation

$$\text{Rate} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Rate} = \frac{3}{x}$$

Although we generally solve math problems without including the units, we must consider them. If the distance is given in "miles," and the time is given in "hours" then the rate is expressed in "miles per hour."

Therefore, the rate is really $\frac{3}{x}$ miles per hour.

Please proceed to question 4 which follows.

.....

Question 4

If the rate of a boat in still water is r miles per hour, and the rate of the current is 3 miles per hour, choose the rate in miles per hour at which the boat travels downstream.

(A) $r - 3$

(B) $r + 3$

(C) $3 - r$

(D) $\frac{3}{r}$

48
1

Question 16

The tens digit of a two-digit number is one-half the units digit. The sum of the original number and the number with its digits reversed is 99 . Apply your knowledge to find the correct statement.

(A) The sum of the digits is 9 .

(B) The difference between the digits is 4 .

(C) The original number is larger than 49 .

(D) The original number is smaller than 29 .

48
2

Very good.

Your choice is correct. The given equation is,

$$10t + u = 2u + 1$$

The left side of this equation represents the number. The right side of the equation is 1 more than twice the units digit of the number. Thus, in words, this equation states that the number is one more than twice the units digit.

Please proceed to question 16 which follows.

Volume 15 Segment 2 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	27 (Sequence Number)
	54	and	56	04 (Type of Punch Card)
	60	and	62	15 (Volume Number)
	66	and	68	02 (Segment Number)

Your READING ASSIGNMENT for this SEGMENT is page 383-384 . You will now be asked a series of questions to draw your attention to the more important points.

Introductory Note

You know a plane travels faster if it has a tail wind, and a motorboat goes slower it if travels against the current of the water.

In this segment, we will investigate the methods you can use to solve motion problems when the motion is affected by a wind or a current working with or against the direction traveled.

Please proceed to question 1 below.

Question 1

If an airplane flies at the rate of m miles per hour for h hours, what do you recognize as the distance it will travel?

(A) mh miles per hour
(B) mh miles
(C) m + h miles
(D) m + h hours

$\frac{50}{1}$

We do not agree.

Try to visualize what is happening. If necessary make a sketch of a boat sailing in a direction opposite to the current of the river. This boat maintains a speed of 5 miles per hour at the same time that the current works to slow the boat down.

Please return to page $\frac{60}{2}$ and try question 5 again.

.....

$\frac{50}{2}$

Traveling upstream or downstream only results in different rates of traveling. It does not change the distance between the start and the turning around point.

Please return to page $\frac{63}{2}$ and try question 8 again.

51/1

Not quite.

If a plane can fly against the wind at a speed of 300 mph while it is bucking a wind of w miles per hour, how fast could it fly if there was no wind? Now, weren't you asked how fast it could fly with the wind?

Please return to page 66/2 and try question 6 again.

.....

51/2

We do not agree.
Your choice indicates that you realized that the time was 2 hours, but did not multiply the total hourly rate properly.
The plane can travel with a speed of 300 miles per hour without any help. There is a tail wind helping the plane along. This enables the plane to go faster. Keep in mind that rate means the number of miles covered in 1 hour.
If the combined rate for one hour is $t + 300$ then the rate for two hours is not written as $2t + 300$.

Please return to page 67/2 and try question 10 again.

$\frac{52}{1}$

Your choice is correct.

The basic formula relating Distance, Rate, and Time is

$$\text{Distance} = \text{Rate} \times \text{Time}$$

If we solve this relation for Time, we get

$$\text{Time} = \frac{\text{Distance}}{\text{Rate}}$$

This can be derived by using the methods for solving equations,

$$D = RT \qquad \div R$$

$$\frac{D}{R} = T$$

Therefore, the time is $\frac{y}{x}$.

Please proceed to question 3 which follows.

.....

$\frac{52}{2}$

Question 3

If a boy swims 3 miles in x hours, choose his rate of swimming.

(A) $3x$ miles per hour

(B) $\frac{3}{x}$ miles per hour

(C) $\frac{x}{3}$ miles per hour

(D) $3 - x$ miles per hour

$\frac{53}{1}$

Do not let all the letters that appear in this question confuse you. Keep in mind the basic formula

Distance = Rate x Time

Furthermore, the total distance traveled at different rates is the sum of the distances traveled at each rate, even if the trips were in opposite directions:

Distance # 1 + Distance # 2 equals the total distance.

Note: Did the problem indicate the actual rate of the wind?

Please return to page $\frac{72}{2}$ and try question 7 again.

.....

$\frac{53}{2}$

The basic formula for this type of problem is

Distance = Rate x Time

However, before you can apply this formula, it is necessary that everything is expressed in the same units. Note that the rate is 440 miles per <u>hour</u>, but the time is expressed in <u>minutes</u>. You should, therefore, change the time to hours.

Please return to page $\frac{74}{2}$ and try question 9 again.

$\frac{54}{1}$

The question asks for the distance that the plane will travel. You know that distance is not measured in hours. May we also suggest that you replace the letter m by some number and the letter h by some number. It will help your thinking.

Please return to page $\frac{49}{2}$ and try question 1 again.

.....

$\frac{54}{2}$

You took a guess, didn't you? The expression 5c means five times the speed of the current. There is no reason for the boat to travel at this speed. The current can merely add or subtract from the speed of the boat depending on the direction it travels.

Please return to page $\frac{60}{2}$ and try question 5 again.

Did you look at your answer to see what it really is?

It says that the rate of the current is divided by the rate of the boat in still water. On what basis can you say that?

Please return to page 47/2 and try question 4 again.

.....

That is definitely not the case.

When one travels downstream, he is aided by the current and the trip takes less time.

Please return to page 63/2 and try question 8 again.

56
1

We cannot agree with you. If a plane can fly 300 miles per hour in spite of bucking a wind, the plane can certainly do better <u>with</u> the wind. Note that

$$(300 - w)$$

is less than 300.

Please return to page 66/2 and try question 6 again.

.....

56
2

We do not agree.

Your choice multiplies the hourly rate of the plane in still air by two, which is fine; but then that answer is multiplied by the hourly rate of the wind for one hour.

That's two mistakes. The two rates in these "upstream-downstream" problems are never multiplied; they are either added or subtracted dependent upon whether the motion is being helped by the current (in the case of downstream) or hindered by it as in the case of upstream.

Please return to page 67/2 and reconsider the problem.

$\frac{57}{1}$

Keep in mind that the distance traveled is the product of rate and time, and not the sum of these quantities.

Please return to page $\frac{74}{2}$ and try question 9 again.

$\frac{57}{2}$

You selected the equation,

$$\frac{25}{10 - x} = \frac{15}{10 + x}$$

Did you solve this equation? You didn't. You should be more curious. Let us solve it now. Multiplying both sides of the equation by the LCD,

$$(10 - x)(10 + x)$$

we get:

$$25\,(10 + x) = 15(10 - x) \qquad [\ D$$

$$250 + 25x = 150 - 15x \qquad \swarrow + 15x, - 250$$

$$40x = -100 \qquad \swarrow \div 40$$

$$x = -\frac{5}{2}$$

We get a negative rate, which of course, is not possible. Hence, your choice is incorrect. You apparently associated the distance downstream with the rate upstream and vice versa to find an expression for the time. This is not correct either. Did you construct a chart and did you analyze the relationships carefully?

Please return to page $\frac{78}{2}$ and try question 12 again.

58
1

Almost right. There was one tricky spot in the problem you overlooked. Although the plane traveled in one direction mp miles and the opposite direction for nq miles, the total distance that the plane traveled is not the same as finding how far from the starting point it is. Check to see which question you were asked.

Please return to page 72/2 and reconsider the question.

.....

58
2

You have the theory of setting up the equation correctly, but didn't you mix up which rates related to which distances?
If the times were the same, then the longer distance should be associated with the greater rate.

Please return to page 73/2 and reconsider the problem.

Yes, your choice is correct.

You used the basic formula for motion problems; namely,

$$\text{Distance} = \text{Rate} \times \text{Time}$$

The rate is m miles per hour, and the time is h hours.

Therefore, the distance is mh miles.

Please proceed to question 2 below.

.....

Question 2

If a boat traveled x miles per hour and covered y miles, choose the number of hours it took.

(A) $\frac{y}{x}$

(B) $\frac{x}{y}$

(C) xy

(D) None of these.

$\frac{60}{1}$

Good. You made the correct choice.

When a boat is traveling downstream, it travels in the same direction as the current. Thus it is aided by the current and its rate is the sum of its rate in still water and the rate of the current. Here is a case where the two rates can be added.

Please proceed to question 5 which follows.

.....

$\frac{60}{2}$

Question 5

A boat is traveling upstream at the rate of 5 miles per hour. If the rate of the current is c miles per hour, choose the number of miles per hour in the rate of the boat in still water.

(A) $5 - c$

(B) $c - 5$

(C) $5 + c$

(D) $5c$

$\frac{61}{1}$

We do not agree entirely. The 600 indicates that you realize that the plane travels that far in two hours without the help of the wind. That's correct. Now, when you consider the wind's contribution, you must realize that its rate, too, is helping the plane for <u>two</u> hours. Your choice indicates that the wind helped for only one hour.

Please return to page $\frac{67}{2}$ and reconsider the problem.

.....

$\frac{61}{2}$

The first sentence of the question states that the boat can go 12 miles in 2 hours traveling downstream. That is, the boat can go 6 miles in 1 hour, going downstream. Thus, the combined rate of the boat and the current is 6 miles per hour. Keep this fact in mind.

Please return to page $\frac{70}{2}$ and try question 14 again.

$\frac{62}{1}$

We do not agree.

One of the letters does have the correct answer next to it.

Please reconsider your choice.

Please return to page $\frac{74}{2}$ and try question 9 again.

.....

$\frac{62}{2}$

We do not agree.

Distance divided by rate is equal to time. Thus, the expressions,

$$\frac{25}{10 - x} \text{ and } \frac{15}{10 + x}$$

represent time. If you read the question again, you will note that the time for each trip is the same. Hence, your equation is incorrect for that reason alone. But then there is a second error. You associated the distance downstream with the rate upstream to find the time and the reverse for the time the other way. This confusion can be avoided if you construct a chart and proceed to analyze the facts and relationships carefully.

Please return to page $\frac{78}{2}$ and try question 12 again.

Very good.

Your choice is correct.

If a plane flies m miles for p hours, the distance it will cover is mp miles. The phrase "with the wind" need not be considered here since the rate m miles per hour is the net rate or actual rate over the ground. Similarly, if the plane travels n miles for q hours, the distance it will cover is nq miles. Again, the phrase "against the wind" need not be considered here, because the effect of the wind was included in the net rate over the ground even though they were in opposite directions. The plane altogether flies the sum of these distances.

That is, it flies

$$(mp + nq)$$

miles.

Please proceed to question 8 which follows.

.....

Question 8

If a boat travels upstream for d miles and then returns to its starting point, choose the correct statement.

(A) The total distance covered is 2d miles.

(B) The distance downstream is greater than d miles.

(C) The time downstream is greater than the time upstream.

(D) The rate downstream equals the rate upstream.

64
1

We can best help you if you follow along as we do a similar problem together.

Problem: A boat makes a trip of 20 miles downstream in the same time that it takes the boat to travel 12 miles upstream. If the rate of the current is 4 miles per hour, write the equation which will enable us to find the rate of the boat in still water.

I VARIABLES:

Let x = the rate of the boat in still water, then

$x + 4$ = the rate of the boat downstream, and

$x - 4$ = the rate of the boat upstream.

We now make a chart of the facts as follows:

	R	x	T	=	D
Upstream	$(x - 4)$		$\frac{20}{x - 4}$		20
Downstream	$(x + 4)$		$\frac{12}{x + 4}$		12

The boxes under the Time column are filled in by using the fact that

$$\text{Time} = \frac{\text{Distance}}{\text{Rate}}$$

We are now ready to write the required equation.
Note that the time is the same going each way although the rates and distances are different.

II RELATIONSHIP

Time upstream equals time downstream.

$$\frac{20}{x - 4} = \frac{12}{x + 4}$$

Solving this equation will give us the rate of the boat in still water.

Please return to page 73/2 and try question 11 again.

$\frac{65}{1}$

We do not agree

To help you get started, note that r denotes the rate of the current.

How do we denote the rate of the boat in still water? Yes, the rate is 4r .

Now, make a chart of the facts, thus:

	R	x	T	=	D
Downstream					15
Upstream					15

Fill in the boxes left open and continue.

Please return to page $\frac{83}{2}$ and try question 13 again.

$\frac{65}{2}$

We do not agree.

Did you start the problem by letting x = the rate of rowing in still water

and letting c = the rate of the current?

The rate with the current is (x + c)

and the rate against the current is (x - c)

Now, read the problem again and try to write 2 equations involving x and c. The solution of these equations will give you the correct answer.

Please return to page $\frac{93}{1}$ and try question 16 again.

66
1

Yes, your choice is correct.

The boat is traveling upstream, which means against the current. The boat maintains a speed of 5 miles per hour while traveling this way. The current acts to slow the boat down at the rate of c miles per hour. Therefore, in still wate. the boat could go faster; that is, the boat could travel 5 + c miles per hour.

Please proceed to question 6 which follows.

.....

66
2

Question 6

A plane flies against the wind at 300 miles per hour. The wind is blowing at w miles per hour. Choose the number of miles per hour at which the plane flies with the wind.

(A) 300 - 2w

(B) 300 + w

(C) 300 - w

(D) 300 + 2w

Very good.

You made the correct choice.

You are to be commended on the fact that you noted that the units used must be uniform. Thus, the rate is 440 miles per hour and the time must also be expressed in hours. Now,

$$15 \text{ minutes} = \frac{1}{4} \text{ of an hour.}$$

Hence,

$$\text{Rate} \times \text{Time} = \text{Distance}$$
$$440 \times \frac{1}{4} = 110 \text{ miles}$$

Please proceed to question 10 which follows.

.....

Question 10

A plane with a cruising speed of 300 miles per hour is flying with a tail wind of t miles per hour. Apply your knowledge to find the number of miles it flies in 2 hours.

(A) 600t

(B) 600 + t

(C) 2t + 300

(D) None of these.

$\frac{68}{1}$

The numbers 25 and 15 represent distances.

The expressions (10 + x) and (10 - x) represent rates of travel.

The quotient of distance over rate expresses time. Read the question again and note what is being said about the time of each trip.

You apparently added a relationship that was not given in the problem.

Please return to page $\frac{78}{2}$ and try question 12 again.

$\frac{68}{2}$

We do not agree.

Surely, you came to the following conclusions:

(1) The plane can fly 210 + 30 = 240 miles per hour with the wind.

(2) The plane can fly 210 - 30 = 180 miles per hour against the wind.

(3) If the plane flies a certain distance and then returns to where it started, the distance going is equal to the distance returning.

(4) Furthermore, we know that the round trip takes $3\frac{1}{2}$ or $\frac{7}{2}$ hours, and this amount of time is spent partly in going and the rest in returning.

Can you now set up a chart of the facts and write the equation that will solve this problem?

Please return to page $\frac{85}{1}$ and try question 17 again.

Such a situation can only be true if the water is perfectly still. However, since the question uses the words downstream and upstream, it is understood that the body of water is in motion in the downstream direction.

Please return to page 63/2 and try question 8 again.

.....

Mary is younger than Ann.
Therefore, m is less than a .
Solve the equation you chose for m and see what you get.

Please return to page 97/1 and try question 1 again.

$\frac{70}{1}$

Very good. You made the correct choice. Let us do this problem together.

I VARIABLES:

r = the rate of the current,

$4r$ = the rate of the boat in still water.

Here is a chart of the facts:

	R	x T	= D
Downstream	$4r + r$	$\frac{15}{4r + r}$	15
Upstream	$4r - r$	$\frac{15}{4r - r}$	15

To fill in the above chart, we note that the distance traveled is the same in either direction; namely, 15 miles. Furthermore, time is equal to distance divided by rate.

II RELATIONSHIP:

Since we are told that the total time spent traveling is 4 hours, we have the equation,

$$\frac{15}{4r - r} + \frac{15}{4r + r} = 4$$

It is always advisable to continue on with the solution so that you can check your math.

$$\frac{15}{3r} + \frac{15}{5r} = 4 \qquad \times\ 15r$$

$$5(15) + 3(15) = 4(15r)$$

$$75 + 45 = 60r \qquad [\ C$$

$$120 = 60r \qquad \div\ 60$$

$$2 = r$$

III CHECK:

	R	• T	D
D	10	$(\frac{15}{10})$	15
U	6	$(\frac{15}{6})$	15

$$\frac{15}{10} + \frac{15}{6} \stackrel{?}{=} 4$$

$$1.5 + 2.5 = 4$$

Please proceed to question 14 which follows.

$\frac{70}{2}$

Question 14

Traveling downstream a boat can go 12 miles in 2 hours. It takes 6 hours for the return trip. Apply your knowledge to find the correct statement.

(A) The boat travels 6 miles per hour in still water.
(B) The rate of the current is 2 miles per hour.
(C) The rate of the current is 4 miles per hour.
(D) The rate of the boat in still water is 2 miles per hour.

$\frac{71}{1}$

Let us see if your answer checks.

If we let c equal the rate of the current, then (you say) the rate of rowing in still water is $(c + 4)$. . Hence, the rate with the current must be $c + (c + 4)$, and the rate against the current must be $(c + 4) - c = 4$. Now, traveling against the current for $\frac{1}{2}$ hour at the rate of 4 miles per hour, the crew will advance only 2 miles. But $\frac{2}{3}$ of 6 miles is 4 miles. Hence, your answer does not check.

Please return to page $\frac{93}{1}$ and try question 16 again.

.....

$\frac{71}{2}$

You let yourself get confused.

Your choice states that Andy is 25 years older than Bob.

But, if you read the first sentence of the question again, you will note that this is <u>not</u> the case.

Please return to page $\frac{89}{2}$ and try question 2 again.

72
1

Very good.

You made the correct choice.

The difference in speed between going with the wind and going against the wind is twice the rate of the wind.

To state this algebraically, suppose we let

r = the rate of the airplane in still air,

w = the rate of the wind.

Then the rate with the wind is

$(r + w)$

and against the wind

the rate is $(r - w)$

The difference between the two rates is $2w$.

Now if the rate against the wind is

$r - w = 300$ $\quad\not\quad + w$

the rate in still air is $r = 300 + w$ $\quad\not\quad + w$

then the rate with wind is

$r + w = 300 + 2w$

Please proceed to question 7 which follows.

72
2

Question 7

A plane flies m miles per hour with the wind for p hours to its destination. Then it turns around and flies at n miles per hour for q hours against the same wind. Apply your knowledge to find the number of miles which the plane flies altogether.

(A) $m + n + p + q$

(B) $(m + p) + (m - p)$

(C) $mp + nq$

(D) $mp - nq$

Yes, "None of these" is the correct answer.
Since the plane has a tail wind that increases its speed, the actual rate of the plane is

$$(300 + t)$$

This expression is the distance the plane covers in 1 hour. Therefore, the plane travels

$$2(300 + t) = (600 + 2t) \text{ miles in } 2 \text{ hours.}$$

This is an example of the Distributive Property.

Please proceed to question 11 below.

Question 11

Apply your knowledge to answer the following: A boat travels 36 miles downstream at the same time that it travels 24 miles upstream. If the rate of the boat in still water is denoted by x and the rate of the current is 3 miles per hour, write the equation which can be used to find x.

(A) $\frac{36}{x - 3} = \frac{24}{x + 3}$

(B) $\frac{36}{x + 3} = \frac{24}{x - 3}$

(C) $24(x - 3) = 36(x + 3)$

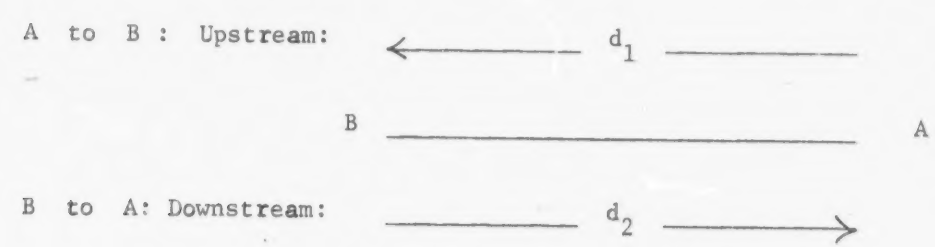
(D) None of these.

74
1

Very good.

Your choice is correct.

The total distance traveled is twice the distance the boat traveled upstream. An examination of the sketch drawn below, makes this fact quite obvious.



Please proceed to question 9 which follows.

74
2

Question 9

If a plane flies at a constant rate of 440 miles per hour for 15 minutes in still air, choose the number of miles it will travel.

(A) 6600

(B) 455

(C) 110

(D) None of these

You seem to be having some trouble with this problem.
If we do a similar problem together, your difficulties may clear up.
Problem: A boat travels 8 miles downstream in 20 minutes, and makes the return trip in 30 minutes. Find the speed of the boat in still water and rate of the current.

I VARIABLE:

Let x = the rate of the boat in still water, and
c = the rate of the current. Then,
$x + c$ = the rate of the boat downstream, and
$x - c$ = the rate of the boat upstream.

Making a chart of the facts, we get:

	R	x	T	=	D
Downstream	$x + c$		$\frac{1}{3}$		8
Upstream	$x - c$		$\frac{1}{2}$		8

Now, the time for each trip is given in minutes, while the rate is given in miles per hour. We, therefore, change 20 minutes to $\frac{1}{3}$ of an hour and 30 minutes to $\frac{1}{2}$ of an hour. The distance traveled is 8 miles each way.

II RELATIONSHIP:

Hence, we have the equations by using the basic formula

$$R \times T = D$$

(1) $\frac{1}{3}(x + c) = 8$ ⟵(3)

(2) $\frac{1}{2}(x - c) = 8$ ⟵(2)

Multiplying the first equation by 3 and the second equation by 2, we obtain

(1') $x + c = 24$

(2') $x - c = 16$

Adding, we get:

$2x = 40$ ⟵ 2

$x = 20$

Substituting, this value is

(1') $(20) + c = 24$ ⟵ 20

$c = 4$

III CHECK

	R	x	D	=	D
D	$(20 + 4)$		$\frac{1}{3}$		8
U	$(20 - 4)$		$\frac{1}{2}$		8

(1) $24 \cdot \frac{1}{3} = 8$

(2) $16 \cdot \frac{1}{2} = 8$

Please return to page 87/1 and try question 15 again.

$\frac{76}{1}$

We do not agree.

You must have confused the rate of the boat with the rate of the current.

Start by letting

x = the rate of the boat in still water, and

c = the rate of the current.

What do you know about the value of $(x + c)$?

What is the rate of the boat going upstream? Try to write 2 equations involving x and c .

Please return to page $\frac{70}{2}$ and try question 14 again.

$\frac{76}{2}$

This problem gave you a little trouble.

It will help if you visualize what is happening. Make a sketch of a river and a toy being carried downstream. Thus:

You, of course, realize that while the swimmer is going after the toy, the toy is continually moving at the rate of the current. Thus, the toy floats for as long as it takes the swimmer to reach it plus the $\frac{1}{2}$ hour start it had. To tell you any more would detract from the fun of your solving this problem. Please continue.

Please return to page $\frac{95}{1}$ and try question 18 again.

$\frac{77}{1}$

If the rate of rowing is less than the rate of the current, the boat will be unable to make the upstream trip. Hence, without any further consideration, you realize that your choice is incorrect.

Please return to page $\frac{93}{1}$ and try question 16 again.

.....

$\frac{77}{2}$

If Mary is younger than Ann, then the letter m representing her age must be less than the letter a representing Ann's age.
Take a second look at your choice, and you will realize that you chose incorrectly. Your choice translated from Algebra states Mary's age is Ann's age subtracted from 5 .

Please return to page $\frac{97}{1}$ and try question 1 again.

$\frac{78}{1}$

Very good. You made the correct choice.

We start as follows:

Let x = the rate of the boat in still water, then

$x - 3$ = the rate upstream and;

$x + 3$ = the rate downstream

We now make a chart of the facts, thus

	R	x	T	=	D
Upstream	$x - 3$		$\frac{24}{x - 3}$		24
Downstream	$x + 3$		$\frac{36}{x + 3}$		36

The fact that $\text{Time} = \frac{\text{Distance}}{\text{Rate}}$

enables us to fill in the boxes under the time column.

II RELATIONSHIP:

Now, we are told that the time upstream is the same as the time downstream, even though the rates and distances are equal.

We, therefore, have the equation,

$$\frac{24}{x - 3} = \frac{36}{x + 3}$$

Solving this equation will give us the speed of the boat in still water.

Please proceed to question 12 which follows.

.....

$\frac{78}{2}$

Question 12

A boat can travel 10 miles an hour in still water. It can travel 25 miles downstream in the same time it takes to go 15 miles upstream. Apply your knowledge to find the equation which expresses the relationships of the problem, if x represents the rate of the stream.

(A) $\frac{25}{10 + x} = \frac{15}{10 - x}$ (B) $\frac{25}{10 - x} = \frac{15}{10 + x}$

(C) $\frac{25}{10 - x} = \frac{15}{10 + x} + 10$ (D) $\frac{25}{10 + x} = \frac{15}{10 - x} - 10$

$\frac{79}{1}$

Your choice, $x = y + 37$ states that Andy is 37 years older than Bob. Did you mean to choose this answer? It is not correct.

Please return to page $\frac{89}{2}$ and try question 2 again.

$\frac{79}{2}$

We do not agree. Let us replace the letters by numbers; it will help you get on the right track.

Suppose the boy was 11 years old 5 years ago; how old will he be 3 years from now? If he were 11 years old 5 years ago, he is now

$$11 + 5 = 16 \text{ years old}$$

and in 3 years, he will be 19 .

Now, return to the letters and express the correct answer algebraically.

Please return to page $\frac{90}{2}$ and try question 4 again.

80
1

Sometimes it helps to do a simple problem which is similar to the one you face and to observe the relationships.

Let us suppose that you are now 16 years old. How old were you 7 years ago? That's right, you were only 9 years old. How did you arrive at this answer?

Please return to page 100/2 and try question 3 again.

80
2

If Walter is now w years old, you correctly state that Walter's age in ten years will be (w + 10). However, Martin ages too, and in 10 years he will be (m + 10) years old.

Please re-read the question and select the correct equation.

Please return to page 110/2 and try question 7 again.

Let us assume for a moment that your choice is correct. The first sentence of the problem states that a boat can travel 12 miles in 2 hours going downstream. That is, the boat travels 6 miles in 1 hour going downstream. If the rate of the boat is 2 miles per hour in still water, then (according to your choice) the rate of the current must be 4 miles per hour. Since the rate of the current is greater than the rate of the boat, the boat could never make the return trip upstream. It is always a good idea to make a common sense check of your answer.

Please return to page 70/2 and try question 14 again.

.....

Let us suppose that you have a younger brother who is now 3 years old, and you are 15 years old. The ratio of your age to your brother's is 15:3 or 5:1 . In 1 year from now, your brother will be 4 and you will be 16 years old. Hence, in 1 year the ratio of your age to your brother's will be 16:4 or 4:1 . You must now realize that the ratio of ages of people changes with time. Find their percentages, then their ages 20 years from now and then determine the ratio.

Please return to page 98/2 and try question 5 again.

$\frac{82}{1}$

We do not agree.

One of the letters does have the correct answer next to it.

Try a direct translation from the English to Algebra.

Please return to page $\frac{97}{1}$ and try question 1 again.

$\frac{82}{2}$

We do not agree.

Start by letting

x = Judy's present age,

$x + 16$ = Sarah's present age.

Now, represent their ages 4 years ago. Remember that each one will be 4 years younger. It will help if you construct a chart.

Read the question again before you attempt to write the correct equation.

Please return to page $\frac{101}{2}$ and try question 8 again.

Very good.

You made the correct choice.

I THE VARIABLES:

We start by setting up a chart of the given facts, as follows:

	R	T	D
Downstream:	$10 + x$	$\frac{25}{10 + x}$	25
Upstream:	$10 - x$	$\frac{15}{10 - x}$	15

The rate of the boat downstream is the sum of its rate in still water and the rate of the current, hence, $(10 + x)$. The rate of the boat upstream is the rate of the boat in still water minus the rate of the stream, hence, $(10 - x)$. The distances traveled in each trip is given as 25 and 15. The Time column is filled in by using the fact that

$$\text{Time} = \frac{\text{Distance}}{\text{Rate}}$$

II RELATIONSHIP:

Reading the question again, we discover that the time for each trip is the same. We, therefore, have the equation,

$$\frac{25}{10 + x} = \frac{15}{10 - x}$$

Please proceed to question 13 which follows.

.....

Question 13

A motorboat can travel in still water 4 times as fast as the rate of the current in a river. If a 15 mile trip upstream and back requires 4 hours, what is the value of r, the rate of the current? Apply your knowledge to find the equation which can be used to find r.

(A) $\frac{15}{4 - r} + \frac{15}{4 + r} = 4$

(B) $\frac{15}{4r - r} + \frac{15}{4r + r} = 4$

(C) $4(4r) = 30$

(D) $4(5r) + 4(3r) = 30$

Very good. Let's review a method for solving.

I VARIABLES:

Let x = the rate of rowing in still water,

c = the rate of the current. Then,

$x + c$ = the rate with the current,

$x - c$ = the rate against the current.

We get the following chart of the given facts:

	R	x	T	=	D
With the current	$x + c$		$\frac{1}{2}$		6
Against the current	$x - c$		$\frac{1}{2}$		4

II RELATIONSHIP: Using the basic formula: R x T = D

we can write these equations:

(1) $\frac{1}{2}(x + c) = 6$ ∢ (2)

(2) $\frac{1}{2}(x - c) = 4$ ∢ (2)

(1') $x + c = 12$

(2') $x - c = 8$ ∢ Add

$2x = 20$ ∢ ÷ 2

$x = 10$

	R	x	T	=	D
With	12		$\frac{1}{2}$		6
Against	8		$\frac{1}{2}$		4

III CHECK:

$x + c = 12$ and

$12 \times \frac{1}{2} = 6$

$x - c = 8$ and

$8 \times \frac{1}{2} = 4$

Substitute in (1')

(1') $x + c = 12$

$10 + c = 12$ ∢ - 10

$c = 2$

Since the rate in still water, 10 mph , is 8 mph more than the rate of the current 2 mph , your choice is correct.

Please proceed to question 17 which follows on page 85/1 .

$\frac{85}{1}$

If a boy were x years old m years ago, he is now m years older. Your answer expresses his present age.

Please return to page $\frac{90}{2}$ and try question 4 again.

.....

$\frac{85}{2}$

Question 17

A plane, which can fly at 210 miles per hour in still air, carries enough fuel for $3\frac{1}{2}$ hours of flying. If it takes off and flies against a 30 mile an hour wind, how far out can it fly and still return safely? Apply your knowledge to find which statement is correct.

(A) It must start back after 1 hour.

(B) It must start back after 300 miles.

(C) It must start back after $1\frac{1}{2}$ hours.

(D) It must start back after 360 miles.

We agree.

You could have solved this problem as follows:

I VARIABLES:

x = the rate of the boat in still water, and

c = the rate of the current, then

$x + c$ = the rate going downstream; and

$x - c$ = the rate going upstream.

II RELATIONSHIP:

This problem hardly requires making a chart of the facts. We will try it without a chart this time. The boat can go 6 miles in 1 hour downstream. Therefore, we can write the equation,

$$(1) \quad x + c = 6$$

The return trip of 12 miles upstream takes 6 hours. Hence,

$$(2) \quad 6(x - c) = 12 \qquad [D$$

$$x - c = 2$$

We have the 2 equations

$$(1') \quad x + c = 6$$

$$(2') \quad x - c = 2 \qquad \text{add}$$

$$2x = 8 \qquad \div 2$$

$$x = 4$$

Substituting in

$$x + c = 6$$

$$4 + c = 6 \qquad - 4$$

$$c = 2$$

Thus, the rate of the current is 2 miles per hour. However, let's check it with a chart. Your choice, the rate of the current, is 2 mph is, therefore, correct.

III CHECK:

	R	x	T	=	D
D	(6)		2		12
U	(2)		6		12

Proceed to question 15 which follows on page 87 .

Question 15

An ice skater skates 10 miles against the wind in $2\frac{1}{2}$ hours. If it takes him only 30 minutes to return, apply your knowledge to find which statement is correct.

(A) His rate of skating in still air is 4 miles an hour.

(B) His rate of skating in still air is 20 miles an hour.

(C) The rate of the wind is 4 miles an hour.

(D) The rate of the wind is 8 miles an hour.

Not quite. Let us examine your choice,

$$p = 2(r - 1) + 3$$

If we subtract 3 from both sides of this equation, we get

$$p - 3 = 2(r - 1)$$

What does the above equation say? The expression $(p - 3)$ is Paul's age 3 years ago. The expression $(r - 1)$ is Rhoda's age last year. Your equation states that 3 years ago Paul was twice as old as Rhoda was last year. This is not the statement of the question.

Please return to page 105/2 and try question 6 again.

Very nice. You solved a pretty difficult problem correctly.
You could have proceeded as follows:

I VARIABLES:

Let t = the time it takes the swimmer to reach the toy.

The toy will have floated $\frac{1}{2}$ hour longer than t. Hence, the toy, floating at the same rate of speed as the current, will have covered a distance of

$$2(\frac{1}{2} + t)$$

The swimmer has to cover the above distance in order to reach the toy. Organize the facts in chart form. This will help you to see the relationships.

	R	x	T	=	D
Toy	2		$t + \frac{1}{2}$		$2(t + \frac{1}{2})$
Swimmmer	$(3 + 2)$		t		$5(t)$

II RELATIONSHIP

The distances traveled downstream by the toy and the swimmer are the same:

$5t = 2(t + \frac{1}{2})$ [D

$5t = 2t + 1$ ∝ - 2t

$3t = 1$ ∝ ÷ 3

$t = \frac{1}{3}$ hour. This is the time it took to catch the toy.

$\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6}$

$= \frac{5}{6}$

III CHECK THIS IN THE CHART:

	R	T	=	D
T	2	$\frac{5}{6}$		$\frac{5}{3}$
S	5	$\frac{1}{3}$		$\frac{5}{3}$

So we now know that the swimmer took $\frac{1}{3}$ of an hour to go downstream $\frac{5}{3}$ miles.

Part II: To return to the dock, his rate was decreased by the current.

	R	T	=	D
Upstream	$(3 - 2)$	$\frac{5}{3}$		$\frac{5}{3}$

The swimmer went 1 mile per hour, and he had to cover $\frac{5}{3}$ miles.

It, therefore, took him $1\ \frac{2}{3}$ hours to return.

Since $1\frac{1}{2} < 2 < 2\frac{1}{2}$, your answer choice is correct.

This is the end of the SEGMENT. Do your HOMEWORK ASSIGNMENT questions, 5, 6, 7, and 8.

Yes, your choice is correct.

Since Mary is 5 years younger than Ann , the letter m must be 5 less than the letter a . Thus,

$$m = a - 5$$

Please proceed to question 2 which follows.

Question 2

Andy is 12 years older than Bob.

Let x represent Andy's age 25 years from now, and y represent Bob's age at that time.

Choose the correct equation to describe the relationship of their ages if both are still alive in 25 years.

(A) $x = y + 25$

(B) $x = y + 12$

(C) $x = y + 37$

(D) None of these.

$\frac{90}{1}$

Yes, your choice is correct.

The man is now 52 years old. Five years ago he was 5 years younger; that is, his age was 52 years less 5 years. Similarly y years ago, the man was y years younger than he now is.

He was (52 - y) years old.

This problem asked his age y - 2 years ago.

Therefore,

$$52 - (y - 2)$$

or $$52 - y + 2$$

$$54 - y$$ is the answer.

Please proceed to question 4 which follows.

$\frac{90}{2}$

Question 4

A boy was x years old m years ago.

Choose his age 3 years from now.

(A) $x - m + 3$

(B) $x + m$

(C) $x + m + 3$

(D) $m + 3$

If Martin is now m years old, then in 10 years, he will be 10 years older; that is, he will be (m + 10) years old. Now, twice this expression is <u>not</u> (2m + 10) .

Please return to page 110/2 and try question 7 again.

We do not agree.

Let us help you get started. If we let

x = Frank's age now,

how do we represent Jerry's present age?

You remembered.

We represent his age by

(27 - x)

Now, answer the following questions: How old will Frank be in 2 years? How old was Jerry 1 year ago? Please read the question again and write the equation which will solve this problem. Don't forget to check your answer.

Please return to page 117/2 and try question 9 again.

Very good. You made the correct choice.

We will enjoy doing this problem together.

I VARIABLES:

Let x = the rate of the skater in still air,

w = the rate of the wind. Then

$x + w$ = the rate of skating with the wind, and

$x - w$ = the rate of skating against the wind.

Making a chart of the facts, we get:

	R	•	T	=	D
With the wind	$x + w$		$\frac{1}{2}$		10
Against the wind	$x - w$		$\frac{5}{2}$		10

Note that the 30 minutes was changed to $\frac{1}{2}$ hour and $2\frac{1}{2}$ hours was changed to $\frac{5}{2}$ hours in order to maintain the same units throughout.

We obtain the equations,

II RELATIONSHIP

(1) $\frac{1}{2}(x + w) = 10$ ‹· (2)

(2) $\frac{5}{2}(x - w) = 10$ ‹· $(\frac{2}{5})$

(1)' $x + w = 20$

(2)' $x - w = 4$ ‹ add

$2x = 24$ ‹÷ 2

$x = 12$

Substitute in (1)'

$x + w = 20$

$12 + w = 20$ ‹ − 12

$w = 8$

III CHECK

$x + w = 20$

$x - w = 4$

R	x	T	=	D
20		$\frac{1}{2}$	=	10
4		$\frac{5}{2}$	=	10

(1) $20 \times \frac{1}{2} = 10$

(2) $4 \times \frac{5}{2} = 10$

Therefore, your choice, the rate of the wind is 8 mph , is correct.

Please proceed to question 16 which follows on page $\frac{93}{1}$.

Question 16

A crew can row 6 miles with the current in 30 minutes. Rowing against the current, it can cover only two-thirds of this distance in the same time. Apply your knowledge to find which statement is correct.

(A) Their rate in still water is 8 miles per hour more than the rate of the current.

(B) Their rate in still water is $\frac{3}{2}$ times the rate of the current.

(C) Their rate in still water is 4 miles per hour more than the rate of the current.

(D) Their rate in still water is 8 miles per hour less than the rate of the current.

You either made a careless error or you forgot how to write twice an expression. Assuming the latter, if Rhoda's age 1 year ago is represented by $(r - 1)$ then twice her age is represented by $2(r - 1)$ and not by $(2r - 1)$. Do you see the difference? How could you ever forget the Distributive Law?

Please return to page 105/2 and try question 6 again.

Very good. You made the correct choice. The distance it can fly before turning back is 360 miles. It is worth while to do it again. Did you enjoy doing this problem?

I VARIABLES:

We are told that the plane can fly 210 miles per hour in still air, and that there is a 30 mile per hour wind. Hence, the rate of the plane with the wind is 240 miles per hour and the rate against the wind is 180 miles per hour. Now, the entire trip takes $3\frac{1}{2}$ hours, or $\frac{7}{2}$ hours. If we let:

t = the time it takes going <u>with</u> the wind, then

$\frac{7}{2} - t$ = the time it takes going <u>against</u> the wind.

Thus, we can make the following chart of the facts:

	R	x	T	=	D
With the wind	240		t		$240t$
Against the wind	180		$\left(\frac{7}{2} - t\right)$		$180(\frac{7}{2} - t)$

II RELATIONSHIP:

Since the distances going and returning to the starting point are the same, we can write the equation,

$$240t = 180(\tfrac{7}{2} - t) \quad [\text{ D}$$
$$240t = 90(7) - 180t$$
$$240t = 630 - 180t \quad \text{✓ + } 180t$$
$$420t = 630 \quad \text{✓ ÷ } 420$$
$$t = \frac{3}{2}$$

Therefore, the distance it can fly before returning is

$$240x\,\frac{3}{2} = 360 \text{ miles}$$

III CHECK:

We can verify the various relations.

Check:	R	X	T	=	D
With	240		$\frac{3}{2}$		360
Against	180		2		360
			$3\frac{1}{2}$		

The distances are equal and the total time is $3\frac{1}{2}$ hours.

Please proceed to question 18 which follows on page $\frac{95}{1}$.

Question 18

A child drops a toy in the river from a dock, and it immediately starts floating downstream at the rate of 2 miles per hour. One-half hour later, a swimmer starts out from the same dock after the toy. If the swimmer can swim 3 miles per hour in still water, how long will it take him to bring the toy back to the dock?

Apply your knowledge to select the letter next to the correct answer.

(A) 3 hours and 15 minutes.

(B) More than $3\frac{1}{2}$ hours, but less than 4 hours.

(C) More than $1\frac{1}{2}$ hours, but less than $2\frac{1}{2}$ hours.

(D) More information is needed to solve this problem.

We do not agree.

One of the letters does have the correct answer next to it.

Please reconsider your choice.

Please return to page 89/2 and try question 2 again.

Volume 15 Segment 3 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS 48 and 50 2 8 (Sequence Number)

54 and 56 0 4 (Type of Punch Card)

60 and 62 1 5 (Volume Number)

66 and 68 0 3 (Segment Number)

Your READING ASSIGNMENT for this Segment is pages 385 - 386.

INTRODUCTORY NOTE

You have heard expressions such as: Time flies; he is getting old; she looks young for her age; and the like. People are very much interested in age; sometimes they are more interested in the age of others than in their own age. When the older generation tries to figure out someone's age, they usually refer to events that happened a specified number of years ago.

For example, you may have overhead a conversation like this one: Aunt Betsy was married 12 years ago, and at the time of her marriage, her brother, who is 3 years younger than she is, celebrated his 18th birthday. Therefore, Aunt Betsy must be 33 years old.

In doing age problems in algebra, we use the same principle that was used to figure out Aunt Betsy's age. If we let x = Aunt Betsy's age now, then 12 years ago she was $(x - 12)$ years old. She was then 3 years older than her brother. Thus, she was

$$18 + 3 = 21 \text{ years old}$$

Therefore, we have the

equation: $$x - 12 = 21$$

and $$x = 33$$

You will now be asked a series of questions to draw your attention to the more important points.

Please proceed to the next page. You will enjoy the interesting age problems of this segment.

97/1

Question 1

Mary is 5 years younger than Ann. If m represents Mary's age, while the letter a represents Ann's age, choose the correct equation to represent this relationship.

(A) m - 5 = a

(B) m = 5 - a

(C) m = a - 5

(D) None of these.

97/2

You must have heard the expression "The age of miracles is over."
Your choice contradicts this expression; you still believe in miracles.
You claim that in 10 years from now, Walter will be 10 years younger than he is now.
We are teasing you.
Maybe you don't understand that 10 years "from now" implies 10 years in the future, not 10 years "ago".

Please examine your answer and make a realistic (correct) choice.

Please return to page 110/2 and try question 7 again.

98
1

Very good. You made the correct choice.

The boy was x years old m years ago. He is now (x + m) years old.

In three years, he will be 3 years older than that; hence, his age will be

$$(x + m + 3)$$

Please proceed to question 5 which follows.

98
2

Question 5

The ratio of the ages of a father and son is 6:1 .

If the son is 5 years old, choose the ratio of their ages 20 years from now .

(A) 6:1

(B) 5:1

(C) 4:1

(D) 2:1

You are saying something that is entirely different from the statement of the question. You are saying that Paul's age p is equal to twice Rhoda's age increased by 2. A question like this one should be read carefully and read more than once in order to be completely familiar with what is being asked. Start by representing Paul's age now, and his age in 3 years. Then, represent Rhoda's age now, and her age last year. After you have done all this, read the question again and try to write the correct relationship.

Please return to page 105/2 and try question 6 again.

If a problem seems complicated, read it over again.

You will be surprised to learn how much clearer a problem becomes after you have become thoroughly familiar with the facts. We are concerned with the ages of two people; father and son.

Begin by letting

x = the father's present age,

y = the son's present age.

How do we represent their ages in 5 years?

$x + 5$ = the father's age in 5 years,

$y + 5$ = the son's age in 5 years.

How do we represent their ages 4 years ago?

Please continue.

Please return to page 103/2 and try question 10 again.

100
1

We agree. If Andy is now 12 years older than Bob, there will always be that same difference in age between them. It will still be true in 25 years. Every living person gets 1 year older in 1 year.

Hence, $x = y + 12$

is the correct answer.

Please proceed to question 3 which follows.

100
2

Question 3

A man is 52 years old today.

Choose the correct representation of his age $y - 2$ years ago.

(A) $50 + y$

(B) $y + 54$

(C) $50 - y$

(D) $54 - y$

Very good.
Your choice is correct.

I If w represents Walter's present age, then

$$(w + 10)$$

denotes his age in 10 years.

If m denotes Martin's present age, then

$$(m + 10)$$

denotes his age in 10 years.

Age	Now	In 10 Years
Walter	w	w + 10
Martin	m	m + 10

II We are told two things about their ages

(1) Walter's age is now three times Martin's age

$$w = 3m$$

(2) In 10 years Walter's age will be twice Martin's age then

$$w + 10 = 2(m + 10)$$

Please proceed to question 8 below.

Question 8

Sarah's age exceeds Judy's age by 16 years. Four years ago, Sarah was twice as old as Judy was then. Apply your knowledge to find which statement is true.

(A) Judy is over 21 .

(B) Sarah is over 21 .

(C) Sarah is twice Judy's age

(D) The sum of their ages is 48 .

$\frac{102}{1}$

You weren't thinking.

A long time ago when you did number problems, you were asked to answer similar questions. For example, express as an equation: a number x is 1 less then twice some number y. That can be represented by $x = 2y - 1$.

The question you are asked now is the same, except that we are talking about a numerator and denominator of a fraction.

Please return to page $\frac{109}{2}$ and try question 11 again.

$\frac{102}{2}$

We were given the 2 fractions $\frac{a}{b}$ and $\frac{c}{d}$. Nothing was said about the comparative value of these fractions except that each one was less than one. Hence, it is not justified to say that $\frac{a}{b}$ is less than $\frac{c}{d}$ or that $\frac{c}{d}$ is less than $\frac{a}{b}$.

Please return to page $\frac{123}{2}$ and try question 13 again.

Very good. You made the correct choice.

I If we let

x = Frank's present age, then

$27 - x$ = Jerry's present age.

You recall that if the sum of two numbers is given, and one of the numbers is denoted by a letter; the other number is that sum minus the letter.

	Now	One Year Ago	Two Years Hence
Frank	x		$x + 2$
Jerry	$27 - x$	$26 - x$	

II Frank's age 2 years hence will be 6 times Jerry's age 1 year ago. We, therefore have the equation

$$x + 2 = 6(26 - x) \qquad [\ D$$
$$x + 2 = 156 - 6x \qquad \propto\ + 6x, - 2$$
$$7x = 154 \qquad \propto\ \div 7$$
$$x = 22$$
$$27 - x = 5$$

III CHECK:

In 2 years Frank will be 24 and 1 year ago Jerry was 4 years old. Note that

$$24 = 6 \cdot 4$$

	Now	One Year Ago	2 Years Hence
F	22		24
J	5	4	
	27		

Please proceed to question 10 which follows

Question 10

Five years from now, a father will be 3 times as old as his son. Four years ago, the father was 30 years older than his son was then. Apply your knowledge to find which choice is correct.

(A) 4 years ago the father was 4 times his son's age.

(B) The father is 4 times his son's age now.

(C) In 10 years, the father will be 3 times as old as his son.

(D) None of these.

$\frac{104}{1}$

If $n = p$ and $d = q$ the fraction certainly has the value $\frac{p}{q}$. However, there are many fractions with different numerators and different denominators which have the same value. If the value of a fraction is $\frac{1}{2}$, you cannot state that the numerator is 1; it could have other values. It is only necessary that the denominator have twice the value for the fraction to equal $\frac{1}{2}$.
You can find a choice which gives a better answer to this question.

Please return to page $\frac{115}{2}$ and try question 12 again.

.....

$\frac{104}{2}$

We do not agree. How did you start this problem?
You let

n = the numerator,
d = the denominator.

Now, you are given two distinct facts about this fraction. Each one leads to an equation involving the letters n and d. For example, the first statement tells you that $2n = d + 1$.

Please continue.

Please return to page $\frac{131}{1}$ and try question 14 again.

Your choice is correct.

We are told that the ratio of the father's age to his son's age is 6:1 . That is, the father is now 6 times as old as his son is.

Since the son is 5 years old, the father must, therefore, be

$$6 \times 5 = 30 \text{ years old.}$$

Now, in 20 years, the son will be 25 and the father will be 50 years old. Thus, the ratio of their ages will be

$$50:25 = 2:1$$

Once again, a chart will aid your thinking.

	Age Now	Age in 20 years
F	30	50
S	5	25

Please proceed to question 6 which follows below.

Question 6

If p represents Paul's age and r represents Rhoda's age, apply your knowledge to find the equation which states that: "In 3 years Paul will be twice as old as Rhoda was last year."

(A) $p + 3 = 2(r - 1)$

(B) $p = 2(r - 1) + 3$

(C) $p + 3 = 2r - 1$

(D) $p = 2r + 2$

$\frac{106}{1}$

This is not possible. If Sarah is now twice as old as Judy, then 4 years ago, she was more than twice Judy's age. The ratio of the ages of two people does not remain the same. For example, if you are now 16 and your brother is 8, you are presently twice as old as your brother. However, 4 years ago, you were 12 and your brother was only 4. That is, 4 years ago, you were 3 times as old as your brother was.

Please return to page $\frac{101}{2}$ and try question 8 again.

$\frac{106}{2}$

We do not agree.

The denominator of the given fraction is not an element of this set.

How did you start this problem? You should have started as follows:

Let n = the numerator,

d = the denominator, and

$\frac{n}{d}$ = the fraction.

One equation follows immediately from the first sentence; the value of the fraction is $\frac{2}{5}$. Now, read the second sentence over again slowly, and see if you can write another equation involving n and d. If you solve the two equations simultaneously, you will obtain a value for n and d. Check these values before you make your choice.

Please return to page $\frac{135}{1}$ and try question 16 again.

$\frac{107}{1}$

We do not agree.

Whatever value the fraction $\frac{c}{d}$ has, it will add to the value of $\frac{a}{b}$ and the result $(\frac{a}{b} + \frac{c}{d})$ will be larger than either of the two fractions. If you convert the two fractions to decimals, you will see that the sum of 2 decimals is larger than either one.

Please return to page $\frac{123}{2}$ and reconsider the problem.

.....

$\frac{107}{2}$

We got a different answer than you did. We both must have started the same way; that is, by letting

n = the numerator

d = the denominator, and

$\frac{n}{d}$ = the fraction.

Now, if you read the question again, you will be able to write the following equations:

$$\frac{n}{d + 1} = \frac{5}{8}$$

$$\frac{n - 4}{d} = \frac{2}{5}$$

Please go over your solution of these equations, and when you get an answer; check it.

Please return to page $\frac{126}{2}$ and try question 17 again.

108
1

We can best help you by doing a similar problem together.

Problems

(A) The denominator of a fraction is 5 more than its numerator.

(B) If 3 is added to both numerator and denominator, the value of the fraction becomes $\frac{2}{3}$.

Find the fraction.

Solution:

Let n = the numerator,

d = the denominator of the given fraction. Then,

$\frac{n}{d}$ = the original fraction.

Now, we have (1) $d = n + 5$

Furthermore, if we take the original fraction $\frac{n}{d}$ and add 3 to both numerator and denominator, we obtain the new fraction $\frac{n + 3}{d + 3}$ and this fraction is equal to $\frac{2}{3}$. Thus we can write the equation,

$$(2) \quad \frac{n + 3}{d + 3} = \frac{2}{3}$$

Replacing d by $(n + 5)$ from equation (1), we get,

$$\frac{n + 3}{n + 5 + 3} = \frac{2}{3}$$

$$\frac{n + 3}{n + 8} = \frac{2}{3} \qquad \times 3(n + 8)$$

$$3(n + 3) = 2(n + 8) \qquad [\text{D}$$

$$3n + 9 = 2n + 16 \qquad - 2n, - 9$$

$$n = 7$$

and $$d = n + 5$$

$$d = 12$$

Hence, the fraction is $\frac{7}{12}$.

Check:

(A) The denominator is 5 more than the numerator.

(B) If 3 is added to both the numerator and denominator the value becomes $\frac{2}{3}$.

(A) $12 = 7 + 5$

AND (B) $\frac{7 + 3}{12 + 3} = \frac{10}{15}$

$$= \frac{2}{3}$$

Please return to page 125/1 and try question 15 again.

Very good. You made the correct choice.
Let us do this problem together. There are two parts to the problem and we can use two unknowns. Let

x = the father's age,
y = the son's present age.

Now, in 5 years, the father will be $(x + 5)$ years old and the son will be $(y + 5)$ years old. We are told that at that time, the father will be 3 times as old as the son. We, therefore, obtain the equation,

$$(1)\quad x + 5 = 3(y + 5) \qquad [\ D$$
$$x + 5 = 3y + 15 \qquad \swarrow -3y, -5$$
$$x - 3y = 10$$

Furthermore,

(A) The father's age 4 years ago was $(x - 4)$ and the son's age 4 years ago was $(y - 4)$. We are told that the father was 30 years older than the son at that time.

$$(2)\quad x - 4 = (y - 4) + 30 \qquad [\ C$$
$$x - 4 = y + 26 \qquad \swarrow -y, +4$$
$$x - y = 30$$

(B) We have the following 2 equations in x and y :

$$(1')\ x - 3y = 10 \quad \text{Subtract (2') from (1')}.$$
$$(2')\ \underline{x - y = 30}$$
$$-2y = -20 \qquad \swarrow \div -2$$
$$y = 10$$

Substituting this value of y in the second equation, we have

$$x - (10) = 30 \qquad \swarrow + 10$$
$$x = 40$$

Thus, the father is 40 years old and the son is 10 years old.

Check:

In 5 years the father will be 45, and the son will be 15; and $45 = 3(15)$. Four years ago, the father was 36 and the son was 6; and $36 = 6 + 30$.

Please proceed to question 11 below.

Question 11

Another application of the technique of choosing the variables and setting up the equations is presented in the following questions concerning fractions.

The numerator of a fraction is 1 less than twice the denominator. If n represents the numerator and the denominator is represented by d, choose the correct equation.

(A) $n - 1 = 2d$ (B) $n = 2d - 1$

(C) $n = 2(d - 1)$ (D) $n + 2d = 1$

110
1

We agree. Let us systematically list the given facts. Thus, in chart form,

	Age	Last Year	Now	In Three Years
	Paul	$p - 1$	P	$p + 3$
	Rhoda	$r - 1$	r	$r + 3$

I p = Paul's present age

r = Rhoda's present age

$p + 3$ = Paul's age in 3 years from now

$r - 1$ = Rhoda's age 1 year ago.

II We will now read the question once more. Note that Paul's age in 3 years is equal to twice Rhoda's age 1 year ago. We, therefore, have

$$p + 3 = 2(r - 1)$$

Please proceed to question 7 which follows.

110
2

Question 7

Walter is 3 times as old as Martin. Ten years from now Walter will be twice as old as Martin will be then. If w represents Walter's present age, and m represents Martin's present age, apply your knowledge to select the second equation which fits the problem if the first equation is $w = 3m$.

(A) $w + 10 = 2m$

(B) $w + 10 = 2m + 10$

(C) $w - 10 = 2(m + 10)$

(D) $w + 10 = 2(m + 10)$

111/1

Did you check your answer?

You didn't bother because you were sure you were right.

We don't agree.

Let's do some examining of your "facts."

If the sum of the ages of Sarah and Judy is 48, then Sarah's age is 32 and Judy's age is 16. Now, 4 years ago, according to your choice, Sarah was 28 and Judy was 12. But 28 is <u>not</u> twice 12.

Please reconsider your choice and this time check your answer.

Please return to page 101/2 and try question 8 again.

111/2

Could you have confused the numerator with the denominator?

Please go over your work, and you will discover the error you made.

Check your answer by referring to the statements in the problem.

Please return to page 131/1 and try question 14 again.

112
1

Your answer does not check.

If $n + 5 = d$

then the fraction $\frac{n}{d}$ would equal

$$\frac{n}{n + 5}$$

The numerator decreased by 1 and the denominator increased by 5 would make this fraction

$$\frac{n - 1}{n + 10}$$

Since the reciprocal of this fraction should equal 5 you would have:

$$\frac{n + 10}{n - 1} = 5$$

$$n + 10 = 5n - 5$$

$$15 = 4n$$

$$? = n$$ [would be a fraction which is not acceptable; therefore, your choice is wrong

Please return to page 135/1 and try question 16 again.

.....

112
2

The set of positive integers (P) and the set of natural numbers (N) are two different names for the same set.

Please return to page 133/1 and try question 1 again.

By making this choice, you said that

$$\frac{a + c}{b + d} \text{ is less than both } \frac{a}{b} \text{ and } \frac{c}{d}$$

Let's take a simple example and examine it.

Suppose the two original fractions were

$$\frac{1}{3} \text{ and } \frac{1}{2}$$

If we combine them in the form you chose, we will get

$$\frac{1 + 1}{3 + 2} \text{ which equals } \frac{2}{5}$$

Now $\frac{1}{3} < \frac{2}{5} < \frac{1}{2}$

It is helpful to convert the fractions to decimal form for easy comparison.

.3.., .4 .5

Therefore, it is not correct to say that

$$\left[\frac{a + c}{b + d} < \frac{a}{b}\right] \text{ and } \left[\frac{a + c}{b + d} < \frac{c}{d}\right]$$

Note: Only one exception is enough to disprove a statement.

Therefore,

$$\frac{a + c}{b + d}$$

is not the least value.

Please return to page 123/2 and reconsider the question.

.....

It is true that zero is not a natural number and must be included in the set of integers. But, you have omitted a whole sub-set of integers; the negative whole numbers.

Please return to page 141/2 and consider question 3 again.

114/1

This problem must have given you a little trouble. However, it is not different from any of the other problems dealing with the numerator and denominator of a fraction. To help you get started,

let x = the present age of P

y = the present age of Q

The ratio of their present ages is the fraction $\frac{x}{y}$. What fraction expresses the ratio of their ages 3 years ago? What fraction expresses the ratio of their ages 7 years from now? The answers to these questions will give you 2 equations in x and y. Please continue.

Please return to page 121/1 and try question 18 again.

.....

114/2

The only time when a fraction is undefined is when the denominator of the fraction is equal to zero. You must, therefore, examine all the choices and find the one which makes the denominator equal to zero.

Please return to page 144/2 and try question 5 again.

Question 12

A fraction has the value $\frac{p}{q}$. Choose the correct statement concerning the numerator n and the denominator d.

(A) $n = px$ and $d = qx$

(B) $\frac{n}{d}$ can equal only $\frac{p}{q}$

(C) $\frac{n}{d} = \frac{p + x}{q + x}$

(D) $\frac{n}{d} = \frac{p \pm x}{q \pm x}$

Yes, your choice is correct.

The numerator n is 1 less than twice the denominator d.

Algebraically, we write,

$$n = 2d - 1$$

Please proceed to question 12 which follows.

116/1

Let us see if your choice checks.

If 3 is the numerator, then by substituting in the first sentence (or first equation) you will get:

$$\begin{aligned} 2(3) &= d + 1 \\ 6 &= d + 1 \\ d &= 5 \end{aligned}$$

Hence, according to your choice, the fraction is $\frac{3}{5}$. Now, if the denominator is decreased by 1, we get

$$\frac{3}{5 - 1} = \frac{3}{4} \quad \text{but } \frac{3}{4} \text{ does not equal } \frac{2}{3}.$$

Thus, your choice is incorrect. Do you see why it is important to check your answer?

Please return to page 131/1 and try question 14 again.

.....

116/2

We do not agree.

One of the letters does have the correct answer next to it.

Please reconsider your choice.

Please return to page 135/1 and try question 16 again.

Yes, Sarah has been eligible to vote for a number of years.
We begin as follows:

	Age Now	Age 4 Years Ago
Judy	x	$x - 4$
Sarah	$x + 16$	$x + 12$

Four years ago, their ages were $(x - 4)$ and $(x + 16 - 4) = x + 12$.
Since Sarah was then twice as old as Judy was, we have the equation,

$$x + 12 = 2(x - 4) \qquad [\ D$$
$$x + 12 = 2x - 8 \qquad \swarrow -2x, -12$$
$$-x = -20 \qquad \swarrow \cdot(-1)$$
$$x = 20$$

and $x + 16 = 36$

Thus, Judy is 20 years old and Sarah is 36.

Check:

	Now	4 Years Ago
J	20	16
S	36	32

Four years ago, Sarah was 32 and Judy was 16; and $32 = 2(16)$.
Please proceed to question 9 which follows.

Question 9

The sum of Frank's age and Jerry's age is 27 years. Frank's age 2 years from now will be 6 times Jerry's age 1 year ago. Apply your knowledge to find which choice is correct.

(A) Frank's age is between 10 and 20.

(B) Jerry's age is between 6 and 10.

(C) Frank is now 5 times as old as Jerry is now.

(D) None of these.

118
1

We agree. The set of positive integers is the set

$$P = \{1, 2, 3, \ldots\}$$

The set of natural numbers is the same set as the above. Since the set of natural numbers is infinite, the set of positive integers is likewise infinite.

It is infinite for the simple reason that there can't be a "largest number" since whatever might be designated as the "largest" can always be made larger by adding at least "one" to it.

Please proceed to question 2 which follows.

.....

118
2

Question 2

If p and q represent positive integers, which of the following (operations on p and q) do you recognize as always resulting in a positive integer?

I $p - q$

II pq

(A) I only

(B) II only

(C) Both I and II

(D) Neither I nor II

119
1

Yes, the set I , the integers, includes both positive and negative whole numbers. However, when you consider the number line going from - to + don't you have to consider zero as one of the integral points?

Please go back to page 141/2 and reconsider the choices.

.....

119
2

Since the set of rational numbers is closed under the four operations--addition, subtraction, multiplication, and division (except division by zero), it is true that your choice represents only rational numbers. However, you have overlooked the fact that there is a better choice.

Please return to page 136/2 and try question 6 again.

$\frac{120}{1}$

Very good. You made the correct choice.
You, of course, started as follows:

Let n = the numerator
d = the denominator
$\frac{n}{d}$ = the original fraction

(A) The first sentence tells us that

(1) $\frac{n}{d+1} = \frac{5}{8}$ ✕ $8(d+1)$

$8(n) = 5(d+1)$ [D

$8n = 5d + 5$ ✕ $-5d$

(1') $8n - 5d = 5$

(B) The second sentence tells us that

$\frac{n-4}{d} = \frac{2}{5}$ ✕ $(5d)$

$5(n-4) = 2d$ [D

$5n - 20 = 2d$ ✕ $-2d, +20$

(2') $5n - 2d = 20$

We, therefore, have to solve the following equations simultaneously:

(1') $8n - 5d = 5$ ✕ (2)
(2') $5n - 2d = 20$ ✕ (-5)

$16n - 10d = 10$
$-25n + 10d = -100$
$-9n = -90$ ✕ Add
$n = 10$ ✕ ÷ - 9

Substitute in (1)

$8(n) = 5(d+1)$ [D
$8(10) = 5d + 5$
$80 = 5d + 5$ ✕ - 5
$75 = 5d$ ✕ ÷ 5
$15 = d$

Check:

(A) If you add 1 to the denominator the fraction = $\frac{5}{8}$

$\frac{10}{15} \longrightarrow \frac{10}{16} = \frac{5}{8}$

(B) If you subtract 4 from the numerator, the fraction

$\frac{10}{15} \longrightarrow \frac{6}{15} = \frac{2}{5}$

Question 18

Apply the proper principle to solve the following problem:

The ratio of the ages of 2 people P and Q , 3 years ago was 5:7 .
The ratio of their ages 7 years from now will be 5:6 .
What are their present ages?

Select the letter next to the correct statement.

(A) Their ages are 25 and 35 years.

(B) The sum of their present ages is 35 .

(C) The difference of their present ages is 4 years.

(D) None of these.

.....

We do not agree.
Zero divided by 1 equals zero, that is,

$$\frac{0}{1} = 0 \qquad \text{Since } 0 = 1 \cdot 0$$

But, 0 is a rational number; hence your choice is incorrect.

Please return to page 128/2 and try question 4 again.

$\frac{122}{1}$

No, you have gone overboard.

The integers are "whole numbers." They include zero as well as positive and negative whole numbers. But the rational numbers are "ratios" and include all of the fractions, only some of which are whole numbers.

e.g.

$$\frac{8}{2} \quad \text{or} \quad \frac{10}{2}$$

Unless the numerator is divisible by the denominator, the rational number is not an integer.

Please return to page $\frac{141}{2}$ and try the question again.

.....

$\frac{122}{2}$

The given fractions have fairly large numerators and denominators. Can these fractions be reduced? Now,

$$\frac{51}{68} = \frac{17 \times 3}{17 \times 4} = \frac{3}{4}$$

Can the fraction $\frac{54}{73}$ be reduced? If either the numerator or the denominator is a prime number, the fraction cannot be reduced.

If 2 fractions cannot be reduced to the same fraction, then they are not equal.

Please return to page $\frac{142}{2}$ and try question 7 again.

Very good. You made the correct choice.

Consider the fractions,

$$\frac{2}{5} \text{ , } \frac{4}{10} \text{ , } \frac{6}{15} \text{ , and so on.}$$

These fractions are equivalent.

$$\frac{2}{5} = \frac{2}{5} \cdot \frac{2}{2} = \frac{4}{10}$$

$$\frac{2}{5} = \frac{2}{5} \cdot \frac{3}{3} = \frac{6}{15}$$

A fraction remains unaltered in value if both the numerator and the denominator are multiplied by the same number.

In general

$$\frac{n}{d} = \frac{n}{d} \cdot 1 \qquad \left[\text{Since } \frac{x}{x} = 1 \text{ we can use and value of } x \text{ .}\right.$$

$$= \frac{n}{d} \cdot \frac{x}{x}$$

Therefore, if $\frac{n}{d} = \frac{p}{q}$

it will equal $\frac{px}{qx}$

Question 13

Relate to the proper principle and answer the following question:

Given that $\frac{a}{b}$ and $\frac{c}{d}$ are positive proper fractions, which of the following is the fraction of least value?

(A) $\frac{a}{b}$

(B) $\frac{n}{d} = (\frac{a}{b} + \frac{c}{d})$

(C) $\frac{a + c}{b + d}$

(D) $\frac{ac}{bd}$

Very good. You made the correct choice.

We start by letting

n = the numerator

d = the denominator of the fraction.

(A) Now, the first sentence tells us that twice the numerator exceeds the denominator by 1. We, therefore, have the equation:

(1) $2n = d + 1$

(B) The second sentence tells us that if the denominator is decreased by 1, the value of the fraction is $\frac{2}{3}$. Thus, the original fraction is $\frac{n}{d}$, and the new fraction is $\frac{n}{d-1}$, and

(2) $\frac{n}{d-1} = \frac{2}{3}$ ∝ $3(d-1)$

$2d - 2 = 3n$ ∝ $+ 2$

$2d = 3n + 2$ ∝ $\div 2$

(2') $d = \frac{3n+2}{2}$

Substituting this value of d in equation (1), we get:

$2n = (\frac{3n+2}{2}) + 1$ ∝ (2)

$4n = 3n + 2 + 2$ ∝ $- 3n$

$n = 4$

Since (2') $d = \frac{3n+2}{2}$

$d = 7$ and the fraction is $\frac{4}{7}$. Hence, $n + d = 11$

Check:

(A) Twice the numerator is 8 and 8 exceeds the denominator 7 by 1.

(B) If the denominator is decreased by 1, $\frac{4}{7}$ becomes $\frac{4}{6}$ which does equal $\frac{2}{3}$.

Please proceed to question 15 which follows on page $\frac{125}{1}$.

Question 15

The denominator of a fraction is 6 more than the numerator.
If 1 is added to each, the value of the resulting fraction is $\frac{1}{2}$.
Apply your knowledge to find the correct statement about the numerator, n and the denominator d .

(A) n is even
(B) n is odd
(C) d is even
(D) d is divisible by 3

.....

We do not agree. If p and q are positive integers, although the sum is always a positive integer, the difference p - q is not always a positive integer. Why not? Suppose p = 5 and q = 8 , then p - q = -3 and -3 is not a positive integer.

The result of subtraction depends on the order as well as the size of the numbers.

$$8 - 5 \neq 5 - 8$$

Please return to page 118/2 and try question 2 again.

Very good. You made the correct choice. We begin as follows:

I VARIABLES:

Let n = the numerator

d = the denominator

$\frac{n}{d}$ = the fraction

II RELATIONSHIP:

The first sentence tells us that

(1) $\frac{n}{d} = \frac{2}{5}$ or $(5n = 2d)$

If the numerator is decreased by 1
and the denominator is increased by 5 ,
the new fraction is

$$\frac{n - 1}{d + 5}$$

The reciprocal of this fraction equals 5 ,
therefore,

(2) $\frac{d + 5}{n - 1} = 5$ ⟨(n - 1)

$d + 5 = 5(n - 1)$ [D

$d + 5 = 5n - 5$ ⟨-5

$d = 5n - 10$ ⟨(2)

$2d = 10n - 20$ [Substitute $(5n = 2d)$

$5n = 10n - 20$ ⟨- 10n

$-5n = -20$ ⟨÷-5

$n = 4$

Substituting

$(n = 4)$ in (1) ,

$2d = 5 \cdot (4)$ ⟨÷2

$d = 10$

III CHECK:

The fraction equals $\frac{2}{5}$

$\frac{n}{d} = \frac{4}{10} = \frac{2}{5}$

Decrease the numerator by 1 and increase the denominator by 5 and the reciprocal = 5

$\frac{4}{10} \rightarrow \frac{4 - 1}{10 + 5} = \frac{3}{15}$

$\frac{15}{3} = 5$

Question 17

If you add 1 to the denominator of a fraction, the fraction becomes equal to $\frac{5}{8}$. If, on the other hand, you subtract 4 from its numerator, it becomes equal to $\frac{2}{5}$. Apply your knowledge to find the denominator of the original fraction.

(A) 39
(B) 23
(C) 7
(D) 15

How can we discover whether two fractions are equal?
One way is to change each fraction to its decimal equivalent.
Thus, for example,

$$\frac{57}{76} = 76\overline{)\,57.00}\quad .75$$
$$532$$
$$380$$
$$380$$

Try the same procedure with the fraction $\frac{54}{73}$ and see what you get.

In order for two fractions to be equal they must have the same decimal equivalent value.

Please return to page $\frac{142}{2}$ and try question 7 again.

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1

That is correct.

The set of integers, I is composed of the positive integers (or natural numbers), zero, and the negative integers.

It includes, therefore, all the whole numbers as the number line from (-) to (+), right through zero.

Now, proceed to question 4 which follows below.

.....

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2

Question 4

Choose which of the following is NOT a rational number:

(A) $\frac{1}{0}$

(B) $\frac{0}{1}$

(C) $1 - 0$

(D) $0 - 1$

You claim that the value of a fraction remains the same if the same number is added to both the numerator and denominator.

Let us show you a counter-example.

Consider the fraction $\frac{3}{9}$, and let us add 3 to the numerator and denominator of this fraction,

$$\frac{3 + 3}{9 + 3} = \frac{6}{12} = \frac{1}{2}$$

Now, let's take the same fraction $\frac{3}{9}$ and subtract 1 from both the numerator and the denominator:

$$\frac{3 - 1}{9 - 1} = \frac{2}{8} = \frac{1}{4}$$

The fraction's value has been changed from $\frac{1}{3}$ to $\frac{1}{2}$ and then to $\frac{1}{4}$.

Thus, in general, a fraction changes its value when the same number is added to both numerator and denominator.

Please return to page $\frac{115}{2}$ and try question 12 again.

You were asked to arrange the fractions in size order from largest to smallest. Now $\frac{a}{b}$ will be greater than $\frac{c}{d}$, if $ad > bc$.

This follows directly from the operation of dividing both sides of an inequality by the same positive value:

It is given that: $ad > bc \quad \left(\div \frac{1}{bd}\right)$

then it follows: $\frac{a}{b} > \frac{c}{d}$

Let's apply this to check one of the combinations included in your choice.

$$\frac{5}{7} \; ? \; \frac{11}{14}$$

$$5 \times 14 \; ? \; 7 \times 11$$

$$70 < 77$$

Therefore, your choice is incorrect.

Now return to page $\frac{148}{2}$ and use this principle to check the other choices.

Very good.

You probably tried a few cases and then generalized. But how do you know that there aren't some cases that don't work that you didn't happen to try?

Of course, you could have reasoned in this fashion: Since a proper fraction is a part of a whole; if a fraction of a fraction is taken, you will have a part of the fraction which would, therefore, be still smaller than the fraction.

But wouldn't you like to see a proof in algebra?

Let's assume that $\frac{a}{b}$ and $\frac{c}{d}$ are proper fractions; that is, $a < b$ and $c < d$ and that we wish to show

$$\frac{ac}{bd} < \frac{a}{b} \qquad < -\frac{a}{b}$$

$$\frac{ac}{bd} - \frac{a}{b} < 0 \qquad [\ C$$

$$\frac{ac}{bd} - \frac{a(d)}{b(d)} < 0$$

$$\frac{ac - ad}{bd} < 0 \qquad [\ \text{Factor}$$

$$\frac{a(c - d)}{bd} < 0$$

Since $c < d$, $(c - d)$ is negative and this last statement is true, verifying our assumption.

Please go on to question 14 on page $\frac{131}{1}$.

Question 14

Twice the numerator of a fraction exceeds the denominator by 1 . If the demoninator is decreased by 1 the resulting fraction has the value $\frac{2}{3}$. Apply your knowledge to find the correct statement about the numerator n and the denominator d .

(A) $n + d = 8$

(B) $n = 7$

(C) $n = 3$

(D) $n + d = 11$

You are partially correct, which in math, means your answer is wrong. If two positive integers (p and q where $p < q$) are subtracted, the answer is negative so that part of your answer is correct. (p - q) does not always result in a positive number.
However, if p and q are multiplied, the answer is always a larger positive number

$$(+p)\ (+q) = +pq$$

You said pq is never positive. That's why you are wrong.

Please return to page 118
2 and choose the correct answer.

Volume 15 Segment 4 begins here:

Obtain a PUNCH CARD from your instructor. In addition to other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48 and 50	2	9	(Sequence Number)
	54 and 56	0	4	(Type of Punch Card)
	60 and 62	1	5	(Volume Number)
	66 and 68	0	4	(Segment Number)

Your READING ASSIGNMENT for this Segment is pages 397 - 403 .

Every time we have to make a giant step forward, it is advisable to look back and strengthen the knowledge obtained thus far. A good background makes for a better understanding of what lies ahead. In the segment after this one, we are going to make a giant step forward and talk about a different kind of number. We must, therefore, take inventory of the different numbers we have been dealing with up to now.

We started with the counting numbers 1 , 2 , 3 , ...

These numbers were also called natural numbers. Our study of algebra then led us into negative whole numbers such as -1 , -2 , -3 , ... Furthermore, we encountered the number 0 quite often. This seemingly larger set of numbers consisting of all the negative whole numbers, zero and the natural numbers is called the set integers, and is denoted by

$$I = \{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

In our further study, we also came across numbers like

$$-\frac{2}{3}\ ,\quad -\frac{4}{5}\ ,\quad \frac{9}{10}\ ,\quad \frac{3}{3}\ ,\quad -\frac{5}{2}$$

This again, seemingly larger set consisting of integers and positive and negative fractions is called the set of Rational Numbers. This set is denoted by

$$R = \left\{ \frac{p}{q} \;\middle|\; (p,q) \in I, \quad q \neq 0 \right\}$$

In this segment, we will refresh our memory about these different numbers and get to understand them better.

Please proceed to page 133 .
1

Question 1

You have learned that the natural numbers go on without end; that is, the set of natural numbers (N) is infinite.

Which do you recognize as true of the set of positive integers (P) ?

(A) $P > N$ There are more positive integers than natural numbers.
(B) $P < N$ There are fewer positive integers than natural numbers.
(C) P is infinite. The number of positive integers is infinite.
(D) P is finite. The number of positive integers is limited.

.....

You know that

$$1 - 0 = 1$$

and 1 is a rational number.

Did you notice that the word NOT was emphasized?

Please return to page 128/2 and try question 4 again.

Very good. You made the correct choice. We will do this question together.

Let n = the numerator,

d = the denominator of the given fraction. Then,

$\frac{n}{d}$ = the original fraction.

(A) We are told that the denominator is 6 more than the numerator. Hence, we can write the equation,

(1) $$d = n + 6$$

(B) We are now changing the fraction by adding 1 to both numerator and denominator. The new fraction is

$$\frac{n + 1}{d + 1}$$

and this fraction is equal to $\frac{1}{2}$. Thus, we have

(2) $$\frac{n + 1}{d + 1} = \frac{1}{2}$$

Using the value of d from the first equation we get,

$$\frac{n + 1}{(n + 6) + 1} = \frac{1}{2}$$

$$\frac{n + 1}{n + 7} = \frac{1}{2} \qquad \langle 2 \cdot (n + 7)]$$

$$2(n + 1) = 1(n + 7) \qquad [D$$

$$2n + 2 = n + 7 \qquad \langle - n, - 2$$

$$n = 5$$

We substitute this value in

(1) $$d = n + 6$$
$$d = (5) + 6$$
$$d = 11$$

Therefore, the original fraction is: $\frac{5}{11}$

CHECK:

(A) The denominator is 6 more than the numerator ($11 = 5 + 6$).

(B) The fraction formed by adding 1 to both the numerator and denominator equals $\frac{1}{2}$.

$$\frac{5}{11} \longrightarrow \frac{5 + 1}{11 + 1} = \frac{6}{12} = \frac{1}{2}$$

Please proceed to question 16 which follows on page $\frac{135}{1}$.

Question 16

The reduced value of a fraction is $\frac{2}{5}$. If the numerator n is decreased by 1 and the denominator d is increased by 5 , the reciprocal of the result equals 5 . Apply your knowledge to find the correct statement.

(A) $n \in \{4, 6, 8, 10\}$

(B) $d \in \{2, 4, 6, 8\}$

(C) $n + 5 = d$

(D) None of these.

.....

No. There are two fractions that are equal.

Try reducing each if possible, and then compare them.

A second method is to convert each fraction to a decimal equivalent and then compare them.

e.g.

$$\frac{5}{8} = \frac{.625}{8)\ 5.000}$$

Please return to page $\frac{142}{2}$ and try problem 7 again.

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1

Since the set of rational numbers is closed under the operation of addition and division (except division by zero), m will always be defined when the denominator is not zero.

When $r = -5$ the denominator will equal zero and then m will be undefined. Hence, this choice is correct.

Please proceed to question 6 which follows.

.....

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2

Question 6

If p and q represent rational numbers and $q \neq 0$ choose which of the following are rational numbers.

I: $p = q$ II: $p - q$ III: pq IV: $\frac{p}{q}$

(A) I and III only
(B) II and IV only
(C) I, II, III only
(D) I, II, III, IV

You were asked to arrange the fractions in order of size from largest to smallest.

Now $\frac{a}{b}$ will be greater than $\frac{c}{d}$

if $ad > bc$

This follows directly from the operation of dividing both sides of an inequality by the same positive value.

If it is given that $ad > bc$ $\not\div bd$

then $\frac{a}{b} > \frac{c}{d}$

Let's apply this to one of the combinations you chose.

$$\frac{11}{14} \quad ? \quad \frac{7}{8} \qquad \not\cdot (8 \cdot 14)$$

$$8 \cdot 11 \quad ? \quad 7 \cdot 14$$

$$88 \quad < \quad 98$$

Therefore, your choice is incorrect.

Now return to page $\frac{148}{2}$ and use this principle to check the other choices.

When the same number keeps repeating in the quotient of a decimal division problem, we can't keep writing it forever. We solve the problem by using a bar (or vinculum) over the number that repeats (called the repetend). The symbol $.\overline{16}$ means that the digits 16 keep repeating.

When you perform the division

$$6\overline{)1.0000\text{---}}$$

do you get $.161616\text{---}$?

Please return to page $\frac{158}{2}$ and try question 11 again.

Very good. You made the correct choice. We will do this interesting problem together.

Let x = the present age of P ,

y = the present age of Q , then

$\frac{x}{y}$ = the ratio of their present ages.

(A) Three years ago, the ratio of their ages was 5:7 ; that is,

(1) $\frac{x - 3}{y - 3} = \frac{5}{7}$ ≮ $7(y - 3)$

$7(x - 3) = 5(y - 3)$ [D

$7x - 21 = 5y - 15$ ≮ $- 5y , + 21$

(1') $7x - 5y = 6$

In 7 years, the ratio of their ages will be 5:6 ; that is,

(2) $\frac{x + 7}{y + 7} = \frac{5}{6}$ ≮ $6(y + 7)$

$6(x + 7) = 5(y + 7)$ [D

$6x + 42 = 5y + 35$ ≮ $- 5y, - 42$

(2') $6x - 5y = - 7$

We, therefore, have to solve the following equations simultaneously:

(1') $7x - 5y = 6$

(2') $6x - 5y = -7$ (1') - (2') (Subtract)

$x = 13$

Substituting this value of x in equation (1')

$7(13) - 5y = 6$

$91 - 5y = 6$ ≮ $+ 5y , - 6$

$85 = 5y$ ≮ ÷ 5

$17 = y$

Hence: Their present ages are 13 and 17 .

	Age Now	3 Years Ago	7 Years From Now
P	13	10	20
Q	17	14	24
		Ratio $\frac{5}{7}$	Ratio $\frac{5}{6}$

Please proceed to page 139/1 .

You have now completed this Segment.
Hand in your PUNCH CARD.

You should now be able to complete the following problems from your HOMEWORK ASSIGNMENT:
Problems 9, 10, 11, and 12

.....

It is true that $0 - 1$ is not a positive number, since

$$0 - 1 = -1$$

However, the set of rational numbers includes positive and negative integers and positive and negative fractions.

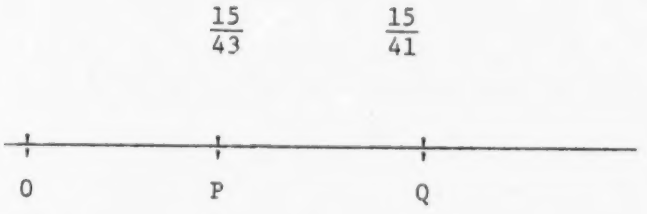
Please return to page 128/2 and try question 4 again.

$\frac{140}{1}$

We do not agree.

Suppose that we graph these numbers as points on the number line.

See the figure below:



Note that the fraction with the larger denominator is the smaller fraction.

Now, ask yourself this question:

How many fractions exist between the points denoted by P and Q ?

Keep in mind that every fraction can be represented by a point on the number line.

Please return to page $\frac{154}{2}$ and try question 9 again.

$\frac{140}{2}$

This choice indicates that the division comes out even.

Let us divide:

$$\frac{3}{11} = 11\overline{)\,3.000}\quad .27$$

$$\frac{22}{80}$$

$$\frac{77}{3}\ \text{remainder}$$

Since we have a remainder of 3 , we cannot agree with your choice.

Please return to page $\frac{164}{2}$ and try question 12 again.

If p and q are each positive integers, then the sum $(p + q)$ and the product pq are each positive integers.

We say that the set of positive integers is closed under the operation of addition and multiplication. What do we mean by saying that a set is closed under a specified operation? We mean that given any two numbers of the set, the numbers obtained after performing the specified operation are still numbers of the set. Thus, for example, in the set of positive integers,

$$\{1, 2, 3, 4, 5, \ldots\}$$

if we add any two numbers of the set, the sum will be a number of the set. The same is true for the product, which is repeated addition. But if we subtract any two members, the answer will be either positive or negative depending upon the relative size of p and q if $p < q$, then $p - q$ will be negative; and we say that the set of positive integers is not closed under subtraction.

Please proceed to question 3 which follows.

.....

Question 3

Recognize what must be added to the set of natural numbers, N, to make the result; the set of integers, I? Choose the letter next to the correct answer.

(A) Zero and the negative integers
(B) Zero
(C) Negative integers
(D) Rational numbers

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1

Since the set of rational numbers is closed under the four operations--addition, subtraction, multiplication, and division (except division by zero), every one of the four quantities offered does represent a rational number. Therefore, this choice is correct.

Please proceed to question 7 which follows.

.....

142
2

Question 7

Apply you knowledge to find which of the following fractions are equal.

I: $\frac{51}{68}$ II: $\frac{54}{73}$ III: $\frac{57}{76}$

(A) I and II
(B) I and III
(C) II and III
(D) None of these.

To convert a rational number $\frac{a}{b}$ to a decimal fraction, you must divide a by b .

When you performed the division

$$6\overline{)\ 1.000\text{--}}$$

did you get .166 and no remainder? You did get a remainder. But you were asked for an equivalent decimal.

Actually	$.166 < \frac{1}{6}$
it is true that	.166
almost equals	$\frac{1}{6}$

but you did not indicate the fact that your answer is incomplete since the places following .166 all contain "6" . A bar (vinculum) over the repeating number will solve the difficulty. Therefore, this choice is not correct.

Please return to page 158/2 and try question 11 again.

We can best help you by doing a similar problem together. There is a clever device to handle the infinitely repeating part of the number.

Problem: If $.\overline{36}$ is changed to a common fraction in lowest terms, find its numerator.

Solution: Now $.\overline{36}$ means .363636...

We proceed as follows:

	Let	N	$= .363636...$	(100)
(2)	then	$100N$	$= 36.363636...$	Subtract line (1) from (2)
(1)		N	$= .363636...$	
		$99N$	$= 36$	$\div 99$
		N	$= \frac{36}{99}$	[Reduce
The infinitely repeating parts drop out when subtracting, leaving finite numbers.			$= \frac{4}{11}$	

Use the same procedure in doing question 14 .

Please return to page 162/2 and try question 14 again.

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1

We agree.

The expression $\frac{1}{0}$ is not a rational number.

In fact, it is not a number. Have you ever wondered why division by zero is forbidden? If division by zero was allowed, we could prove that all numbers are equal . As an illustration, examine the proof that $2 = 1$

Proof:

Given that $a = b$

then $a \cdot a = a \cdot b$

or $a^2 = ab$

Subtract b^2 from both sides of the equality, and we get

$$a^2 - b^2 = ab - b^2$$

Factoring, we get

$$(a - b)(a + b) = b(a - b)$$

Dividing both sides by $(a - b)$ we have

$$a + b = b$$

and since $a = b$ we get

$$2b = b$$

$$2 = 1$$

What did we do wrong? We divided by $(a - b)$ which equals zero, since a is equal to b . Do you see what happens when we divide by zero?

Please proceed to question 5 below.

.....

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2

Question 5

If $$m = \frac{p + q}{r + s}$$

where each letter represents a rational number, apply your knowledge to find which statement would make m undefined.

(A) $p = -q$ (B) $r = -s$

(C) $p = +q$ (D) $r = +s$

We do not agree.

When you write $.2\overline{7}$ it means .27777... with the sevens repeating forever. If we change $\frac{3}{11}$ to a decimal, we get

$$
\begin{array}{r}
.272 \\
11)\ 3.00000 \\
\underline{22} \\
80 \\
\underline{77} \\
30 \\
\underline{22} \\
8
\end{array}
$$

Thus, the third digit after the decimal point is not 7 .

Please return to page 164/2 and try question 12 again.

To convert $\frac{5}{13}$ to a decimal of two places, we must first divide 13 into 5 and carry the answer out to three places. Then we round the three place answer off to two places.

You have done a fine job of division, but did you round off to two decimal places?

Please return to page 175/2 and try question 16 again.

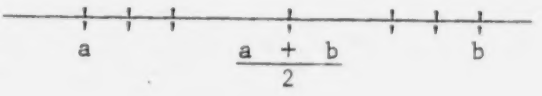
$\frac{146}{1}$

Very good. You made the correct choice.

The fractions $\frac{15}{41}$ and $\frac{15}{43}$ are rational numbers.

Your text makes this very important statement: between any two rational numbers, there exists another rational number. This is equivalent to saying that there is an infinite number of rational numbers having values between any two rational numbers. Let us try to make this point clear.

Suppose that a and b are two rational numbers, and b is greater than a . These numbers can be represented by points on the number line.



Now, if we take the average of the numbers a and b , we obtain the rational number $\frac{a + b}{2}$. On the number line, this average is a point midway between a and b . Similarly, we can find a point midway between $\frac{a + b}{2}$ and b , and this process can be continued indefinitely. Hence, we may conclude that there exists an infinite number of rational numbers between any two rational numbers.

Please proceed to question 10 below.

$\frac{146}{2}$

Question 10

Apply your knowledge to find which of the numbers below fits between $\frac{5}{11}$ and $\frac{8}{17}$ on the number line.

(A) $\frac{6}{12}$

(B) $\frac{6}{13}$

(C) $\frac{6}{14}$

(D) $\frac{6}{15}$

We do not agree. Let us refresh your memory on changing decimals to fractions:

$$4.5 = \frac{45}{10}$$

$$.45 = \frac{45}{100}$$

$$.045 = \frac{45}{1000}$$

and so on.

Please return to page 156/2 and try question 13 again.

Since this decimal is terminating, it represents a rational number and can be written as a common fraction. As a matter of fact, it is equal to

$$\frac{41}{100}$$

Please return to page 163/2 and try question 17 again.

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1

We agree.

Two fractions are equal if when reduced to lowest terms, they are identical.

Thus, the first fraction:

$$\frac{51}{68} = \frac{17 \times 3}{17 \times 4} = \frac{3}{4}$$

The second fraction $\frac{54}{73}$ cannot be reduced because 73 is a prime number.

The third fraction,

$$\frac{57}{76} = \frac{19 \times 3}{19 \times 4} = \frac{3}{4}$$

Thus, only fractions I and III are alike.

Please proceed to question 8 which follows.

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2

Question 8

Apply your knowledge to find which set of numbers are arranged in order of size from largest to smallest.

(A) $\left\{ \frac{5}{7} , \frac{11}{14} , \frac{7}{8} \right\}$

(B) $\left\{ \frac{11}{14} , \frac{7}{8} , \frac{5}{7} \right\}$

(C) $\left\{ \frac{7}{8} , \frac{11}{14} , \frac{5}{7} \right\}$

(D) $\left\{ \frac{7}{8} , \frac{5}{7} , \frac{11}{14} \right\}$

149/1

We do not agree.

You discovered that when you divided the numerator 1 by the denominator 6 that you kept getting a remainder of 6 . You tried to solve this problem by "rounding off" your answer to three decimal places. Ordinarily this gives a good approximation of the value. Actually $.167 > \frac{1}{6}$

However, in this problem you were asked for a decimal with the equivalent (same) value. You must use the bar over the repeating number.

Please reconsider your choice.

Please return to page 158/2 and try question 11 again.

.....

149/2

You were asked to express the fraction in lowest terms.

Please go over your work, and after you have changed $.\overline{45}$ to a fraction, express the fraction in lowest terms.

Please return to page 162/2 and try question 14 again.

150
1

Your choice does not agree with the text that you should have read with greater care. The text specifically states that every rational fraction can be written as a decimal. Either the decimal terminates, as for example the fraction

$$\frac{3}{4} = .75$$

or the decimal is periodic as the fraction

$$\frac{1}{3} = .333...$$

Please return to page 164/2 and try question 12 again.

150
2

Suppose that the given repeating decimal was .44444... instead of .5444... How would you change it to a common fraction? You would proceed as follows:

(1) Let $N = .44444..$

(2) $10N = 4.44444...$

Subtracting (1) from (2) you get

$$9N = 4$$

$$N = \frac{4}{9}$$

Consider now $N = .5\overline{4}$

$$100N = 54.\overline{4}$$

$$10N = 5.\overline{4}$$

The object is to multiply by a power of 10 large enough to isolate the repetend (repeating part) on the right side of the decimal. Then by subtraction, the repetend can be eliminated. Please continue.

Please return to page 169/2 and try question 18 again.

In order to get a decimal correct to two places, it is necessary to calculate the first three decimal places. If the third place is 5 or greater, increase the second place digit by 1 and if the third place digit is less than 5 make no change in the second place digit. Which was the case here? You have gotten an incorrect value.

Please return to page 175/2 and try question 16 again.

.....

You are on the right track.
But is .444 the same as

$$\frac{4}{10} + \frac{4}{100} + \frac{4}{1000} + \text{---} \quad ?$$

Are they both non-terminating?

Please return to page 173/2 and reconsider question 20.

To compare fractions there must be a common basis for determining their relative size.

One method that is particularly good to use when there are many fractions having large denominators is to convert them to decimal fractions.

e.g. $\frac{5}{11} = 11)\overline{5.00}\ (.45 = .45^{+}$

$$\begin{array}{r} .45 \\ 11)\ 5.00 \\ 44 \\ \hline 60 \\ 55 \\ \hline 5 \end{array}$$

$\frac{8}{17} = 17)\overline{8.00}\ (.47 = .47^{+}$

$$\begin{array}{r} .47 \\ 17)\ 8.00 \\ 68 \\ \hline 120 \\ 119 \\ \hline 1 \end{array}$$

Now, you must use this method on the choices given in order to decide which one has a value between $\frac{5}{11}$ and $\frac{8}{17}$.

Please return to page $\frac{146}{2}$ and try question 10 again.

153
1

Since this is already a common fraction, this cannot be the correct answer.

Don't let the minus sign fool you; negative numbers belong to the set of rational numbers.

Please return to page 163/2 and try question 17 again.

153
2

The question specifically asks for the <u>negative</u> square root of 81, not the square root of negative 81.

Please return to page 178/2 and try question 1 again.

154
1

Good. The three fractions you chose are in order of size from largest to smallest. The relationship can be verified by the principle if

$$ad > bc \quad \text{if} \quad \div bd$$

$$\text{then} \quad \frac{a}{b} > \frac{c}{d}$$

$$7 \times 14 \; ? \; 8 \times 11 \qquad 11 \times 7 \; ? \; 14 \times 5$$

$$98 > 88 \qquad 77 > 70$$

$$\text{therefore,} \quad \frac{7}{8} > \frac{11}{14} \qquad \frac{11}{14} > \frac{5}{7}$$

$$\text{therefore,} \quad \frac{7}{8} > \frac{11}{14} > \frac{5}{7}$$

Now proceed to the next question below.

154
2

Question 9

Apply your knowledge to find which statement is true of the numbers between $\frac{15}{41}$ and $\frac{15}{43}$ on the number line.

(A) $\frac{15}{42}$ is the only number between them.

(B) $\frac{15}{42}$ and $\frac{30}{83}$ are the only numbers between them.

(C) There are more than 2 but less than 10 numbers between them.

(D) There are more than 10 numbers between them.

Very good. You made the correct choice.

To change the repeating decimal $.\overline{45}$ to a fraction, we proceed as follows:

Let $N = .454545...$

We must multiply by a power of 10 which is chosen to form a number to the left of the decimal, and at the same time, retain the exact repeating part of the number to the right of the decimal.

Note: If we had multiplied this by 10 the repetends would not have been identical. Therefore,

Number $N = .4545...$ (1) $100N = 45.454545...$

(2) $N = .454545...$

Subtracting (2) from (1) we get,

$$99N = 45$$

$$N = \frac{45}{99}$$

$$= \frac{5}{11} \cdot \frac{9}{9}$$

$$= \frac{5}{11}$$

Please proceed to question 15 which follows below.

Question 15

Perform the process of rounding off $.\overline{53}$ to four decimal places. Select the letter next to the correct answer.

(A) .5300

(B) .5350

(C) .5353

(D) .5354

156
1

Very good. You made the correct choice.

If we carry out the division of the fraction $\frac{3}{11}$ we get

$$
\begin{array}{r|l}
 & .2727\ldots\text{etc.} \\
\hline
11 & 3.00000 \\
 & \underline{22} \\
 & 80 \\
 & \underline{77} \\
 & 30 \\
 & \underline{22} \\
 & 80
\end{array}
$$

That is $\frac{3}{11} = .272727\ldots$

or more briefly $\frac{3}{11} = .\overline{27}$

We indicate the repetend (the part that repeats) by the bar. This is known as a repeating or non-terminating decimal.

Please proceed to question 13 which follows.

156
2

Question 13

Choose the fraction equal to .0045

(A) $\frac{9}{2000}$

(B) $\frac{9}{200}$

(C) $\frac{9}{20}$

(D) None of these

Calculating the value of the fraction in decimal form, you should find the first two digits to be .38 .
This means that

$$.39 > \frac{5}{13} > .38$$

It is still necessary to find the third digit in order to know how to round off. However, in no case does the digit 8 get smaller when rounding off. Therefore, this choice is not correct.

Please return to page 175/2 and try question 16 again.

We do not agree. Let us change each of the terms of the given sum to a decimal. Thus,

$$\frac{4}{10} = .4$$

$$\frac{4}{100} = .04$$

$$\frac{4}{1000} = .004\text{---and so on}$$

Can you express the given sum?

Do you recognize this as a repeating decimal?
Can you find the sum (or the value) of a repeating decimal?

Please return to page 173/2 and try question 20 again.

158
1

Good. When you have many fractions with a variety of denominators that cannot be easily made into fractions with common denominators, the simplest method is to convert all to decimal fractions. (They then will have the same denominator) .

Thus we will have an ordered sequence:

$$\frac{6}{12} > \frac{8}{17} > \frac{6}{13} > \frac{5}{11} > \frac{6}{14} > \frac{6}{15} \qquad \text{since}$$

$$.50 > .47^{+} > .46^{+} > .45^{+} > .42^{+} > .40$$

Therefore your choice $\frac{6}{13}$ does lie between $\frac{8}{17}$ and $\frac{5}{11}$ on the number line.

Now proceed to question 11 below.

.....

158
2

Question 11

Choose the decimal which is equivalent to $\frac{1}{6}$.

(A) $.1\overline{6}$

(B) $.\overline{16}$

(C) .166

(D) .167

Since this is a repeating decimal, it represents a rational number.

$$N = .38\overline{5}$$

(1) $1000N = 385.555...$

(2) $100N = 38.555...$ Subtract line (2) from (1)

$900N = 347.$ $\div 900$

$N = \frac{347}{900}$ Therefore, $.38\overline{5} = \frac{347}{900}$

Therefore, this is not the correct choice.

Please return to page $\frac{163}{2}$ and try question 17 again.

The square root of 81 is not the square root of 9 ; that is

$$\sqrt{81} \neq \sqrt{9}$$

The square root of 81 is 9 . Now you were asked for the negative square root of 81 .

Please return to page $\frac{178}{2}$ and try question 1 again.

Let us illustrate the operation of rounding off.

Consider the number .63548 . If we wish to round this number off to one decimal place, we must choose between either .6 or .7 by deciding which is closer in value.

.7
.63548
.6

We examine the following number 3 . If it is less than 5 as it is in this case, no change takes place; the answer is .6 .

The above number rounded off to two places of decimals is .64 since the third number is 5

.64
.63548
.63

The above number rounded off to four places of decimals is .6355 since the number following the fourth place is 8

.6355
.63548
.6354

Please return to page 155/2 and try question 15 again.

Very good.
You made the correct choice.

If we let

$$N = .\overline{54},\quad 10N = 5.\overline{4},\quad \text{and}\quad 100N = 54.\overline{4}$$

Now, $100N - 10N = 90N$

and $90N = 54.\overline{4} - 5.\overline{4}$

$$90N = 49$$

$$N = \frac{49}{90}$$

and the denominator is 90 .

Please proceed to question 19 which follows.

.....

Question 19

Apply your knowledge to change $.5\overline{24}$ to a common fraction in lowest terms.

Which is the denominator of the fraction?

(A) 999

(B) 99

(C) 330

(D) 990

Very good. You made the correct choice. The decimal

$$.0045 = \frac{45}{10,000}$$

$$= \frac{9.5}{2000.5}$$

Dividing the numerator and denominator by 5 we get

$$\frac{9}{2,000}$$

Please proceed to question 14 which follows.

.....

Question 14

If $.\overline{45}$ is changed to a common fraction in lowest terms, apply your knowledge to find its numerator.

(A) 9

(B) 5

(C) 45

(D) 3

If you calculate the value of $\frac{5}{13}$ as a decimal, you will find that it is a repeating decimal. The value is

$$.\overline{384615}$$

In rounding this off to two places, we note that

$$.39 \; > \; .\overline{384615} \; > \; .38$$

Since the third place digit is 4 , it means that $.\overline{384615}$ is closer to .38 .

Please proceed to question 17 below.

.....

Question 17

Choose the number which cannot be written as a common fraction.

(A) .41

(B) $-\frac{2}{3}$

(C) $.38\overline{5}$

(D) All three can be written as common fractions.

If you perform the division of 1.000--- by 6 the result has the digit 1 followed by an unending string of sixes. That is exactly the meaning of $.1\overline{6}$

Therefore this is the correct answer since

$$\frac{1}{6} = .1\overline{6}$$

$$\text{while} \quad .166 < \frac{1}{6} < .167$$

Please proceed to question 12 which follows below.

.....

Question 12

Apply your knowledge to find which statement is true:

(A) $\frac{3}{11} = .27$

(B) $\frac{3}{11} = .2\overline{7}$

(C) $\frac{3}{11}$ cannot be written as a decimal.

(D) $\frac{3}{11} = .\overline{27}$

We do not agree.

The repeating decimal $.\overline{53}$ is .535353...

Thus, the fourth number after the decimal point is 3 .

To round this off to four decimal places, you must choose between

.5354

.535353

and .5353

You have to examine the fifth number in order to round off to 4 places of decimals.

Please return to page $\frac{155}{2}$ and try question 15 again.

We do not agree.

You found the reciprocal of the square root of 81 .

Instead you were asked for the negative of the square root of 81 .

Please return to page $\frac{178}{2}$ and try question 1 again.

We have to express

$$N = .5\overline{24}$$

as a common fraction.

The general technique is to multiply both sides of the equation by a power of 10 that will leave only the repetend $(\overline{24})$ on the right-hand side.

However, in an expression that contains a mixture of numbers that don't repeat and numbers that do as in $.5\overline{24}$ where the 5 doesn't repeat and the 24 does, it is necessary to multiply once to move the non-repeating part to the left of the decimal, and once again to move the entire combination to the left of the decimal.

This will have the effect of isolating the repetend on the right-hand side. It can be eliminated by subtracting the two derived equations.

Thus,

$$1000N = 524.\overline{24}$$
$$10N = 5.\overline{24}$$

Please continue.

Please return to page 161/2 and try question 19 again.

If the

$$\sqrt{a} = b \quad \text{then} \quad b \cdot b = a$$

If $+5$ is a square root of (-25) then $(+5)(+5)$ should equal (-25). But, this is not the case; hence your choice is incorrect.

Please return to page 184/2 and try question 2 again.

Since the original radical has no sign written in front of it, the quantity

$$\sqrt{9x^{16}}$$

is positive. It is, therefore, incorrect to use both signs.

Please multiply $3x^4$ by $3x^4$ and see what you get. Did you get $9x^{16}$? You did? Then you have forgotten how to multiply letters raised to powers. Please refresh your memory.

$$x^3 \cdot x^3 = x\ x\ x \cdot x\ x\ x$$
$$= x^6$$
$$x^a \cdot x^b = x^{a+b}$$

Please return to page 190/2 and try question 5 again.

Very good.
You made the correct choice.

You, of course, realized that each term of the series is a decimal.

Thus,

$$\frac{4}{10} = .4$$

$$\frac{4}{100} = .04$$

$$\frac{4}{1000} = .004 \quad \text{and so on.}$$

Hence,

$$\frac{4}{10} + \frac{4}{100} + \frac{4}{1000} + \ldots$$

is the same as the sum

$$\begin{array}{l} .4 \\ .04 \\ .004 \\ . \\ . \\ . \\ \hline .4444444\ldots \end{array}$$

Thus, the question reduces to expressing the above repeating decimal as a common fraction. We let

$$N = .444\ldots$$

Actually

(1) $N = .\overline{4}$ ‹·(10)

(2) $10N = 4.\overline{4}$ [Subtract (1) from (2)

$$9N = 4$$

$$N = \frac{4}{9}$$

You have now completed this Segment.
You should now complete the following problems from your HOMEWORK ASSIGNMENT: Problems 13, 14, 15 and 16 .

We agree.

Every terminating decimal can be written as a common fraction.
Every repeating decimal can be written as a common fraction.

$.41$ is a terminating decimal $= \frac{41}{100}$

$.38\overline{5}$ is a repeating decimal $= \frac{347}{900}$

$-\frac{2}{3}$ is already a common fraction.

Please proceed to question 18 which follows below.

.....

Question 18

Apply your knowledge to change $.5\overline{4}$ to a common fraction in lowest terms.
Which is the denominator of that fraction?

(A) 11

(B) 90

(C) 100

(D) 50

170
1

The decimal is indeed $.\overline{53}$ which means

.53535353...

It does not mean .53

which could be written as .530

.5300

.53000 etc.

Therefore, this choice is not correct.

Please return to page 155/2 and try question 15 again.

170
2

It is true that a positive number has two square roots, but the symbol

$$\sqrt{N}$$

has only one value. $\sqrt{N}$ means the principal square root, the positive value only.

Please return to page 189/2 and try question 3 again.

We do not agree.

You didn't simplify $\sqrt{x^{16}}$

It does have a square root. Also, have you forgotten that the symbol $\sqrt{N}$ means only the positive value?

Please return to page 190/2 and try question 5 again.

We do not agree.

The symbol $\sqrt{\ }$ alone does not make the number irrational. If the radicand cannot be factored into two equal factors then it is said to be irrational. The square root of 4 is 2 since $4 = 2 \cdot 2$. We both know that 2 is a rational number.

Please return to page 192/2 and try question 9 again.

172
1

You were asked to find the positive square root of -25.

Since your choice is a negative number, it cannot be right. Besides if

$$\sqrt{a} = b \text{ then } b \cdot b = a$$

and your choice, $(-5) \times (-5) \neq -25$

So it is not the square root of -25.

Please return to page 184/2 and try question 2 again.

172
2

We do not agree.

Let's consider a similar problem, and then you can apply the principle to your question

$$\sqrt{\frac{16x^2}{25y^6}} = \frac{\sqrt{16x^2}}{\sqrt{25y^6}} = \frac{4x}{5y^3}$$

We take the square root of the numerator and denominator separately. The square root of an exponential form such as

$$(b)^n \text{ is } (b)^{\frac{n}{2}}$$

Please return to page 180/2 and try question 6 again.

Very good.

You made the correct choice.

We start by letting

$$N = .5\overline{24}$$
$$10N = 5.\overline{24}$$
$$1000N = 524.\overline{24}$$

Subtracting 10N from 1000N we get

$$990N = 519 \qquad \text{÷ } 990$$
$$N = \frac{519}{990} \qquad \left[\frac{173 \cdot 3}{330 \cdot 3}\right. \text{ Factored}$$
$$N = \frac{173}{330}$$

Notice that in order to get the numbers to the right of the decimal to be exactly alike in two successive equations, it was necessary to shift the decimal point by two places so that the repeating block 24 would drop out in the subtraction.

Please proceed to question 20 which follows.

.....

Question 20

Relate to the proper principle to find the value of the sum

$$\frac{4}{10} + \frac{4}{100} + \frac{4}{1000} + \ldots ,$$

if the addition of this series of numbers goes on indefinitely.

Select the letter next to the correct answer.

(A) .444 (B) $\frac{12}{1000}$

(C) $\frac{4}{11}$ (D) $\frac{4}{9}$

If 25 is one of the factors of 1225 what is the other factor?
If we divide 1225 by 25 we get 49 .

Thus,

$$1225 = 25 \times 49$$

Now apply the rule: $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

Please return to page 183
2 and try question 4 again.

.....

Did you notice the small number 3 written inside the edge of the symbol $\sqrt{}$? For example: $\sqrt[3]{}$

The presence of the number 3 in that place means the cube root of the number inside the radical. If a number is written as the product of three equal factors, one of these factors is the cube root of the number.
What three equal factors equal 64 when multiplied?

Please return to page 186
1 and try question 7 again.

Very good.
You made the correct choice.

The decimal $.\overline{53}$ is indeed .535353...

If we round off to four places, we drop everything from the fifth digit on.
The choice is between .5354 and .5353

.5354
.53535353
.5353

Since the fifth digit is 5 we know it is closer to the higher number. We increase the previous digit by 1 . Therefore, the correct answer is .5354

Please proceed to question 16 which follows.

.....

Question 16

Perform the calculation of $\frac{5}{13}$ correct to two decimal places.

Select the letter next to the correct answer.

(A) .385

(B) .39

(C) .38

(D) .37

Volume 15 Segment 5 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48 and 50	3	0	(Sequence Number)
	54 and 56	0	4	(Type of Punch Card)
	60 and 62	1	5	(Volume Number)
	66 and 68	0	5	(Segment Number)

Your READING ASSIGNMENT for this Segment is:
Pages 403 - 405 , 407 - 409 .
You will now be asked a series of questions to draw your attention to the more important points.

INTRODUCTORY NOTE

In the last segment, we dealt with a large set of numbers; namely, the set of Rational Numbers. You probably thought that you have met up with every possible type of number. As a matter of fact, you have already met up with such a number when you worked with areas and circumferences of circles. Yes, the number π is a different kind of number; it is not a member of the set of Rational Numbers. Furthermore, numbers like $\sqrt{2}$, $\sqrt{5}$, $\sqrt{7}$, $\sqrt[3]{2}$, $\sqrt[3]{9}$ and the like belong to a new set of numbers.

We call these numbers irrational. This does not mean that these numbers are "unreasonable." Irrational means not expressible as the ratio of two integers. We cannot give you a more detailed definition of irrational numbers at this point; you will certainly hear more about these numbers in your future work in mathematics.

Please proceed to page 177/1 .

Your text book illustrates how to approximate the square root of a number to many places of decimals. Since you may find the explanation in the book a little difficult to follow, we are asking you to study the following examples. They should make the procedure explained in the book clearer.

Illustration 1

Find the square root of 20 correct to 2 decimal places.

Solution:

Our first guess is:	$20.00 = 4.00$		
We test this guess:	$\frac{20.00}{4.00} = 5.00$		
Our second guess is:	$\frac{1}{2}(4.00 + 5.00) = 4.50$		
We test this guess:	$\frac{20.00}{4.50} = 4.44$		(to the nearest hundredth)
Our third guess is:	$\frac{1}{2}(4.50 + 4.44) = 4.47$		
We test this guess:	$\frac{20.00}{4.47} = 4.47$		(to the nearest hundredth)

Hence, the square root of 20 to the nearest hundredth is 4.47.

Note that we stopped when the quotient and the divisor were equal.

Please proceed to page $\frac{178}{1}$.

$\frac{178}{1}$

Question 1

Choose the negative square root of 81 .

(A) $\sqrt{-81}$

(B) -9

(C) $-\sqrt{9}$

(D) $\frac{1}{9}$

$\frac{178}{2}$

This mistake is one indicating some confusion on your part. Just as you wouldn't say

$\frac{1}{2}$ of 100 equals $\frac{1}{2}$ of 50

but rather just 50 . The square root of 49 is seven, not the square root of seven. Besides the symbol $\sqrt{49}$ means only the positive square root.

Please return to page $\frac{189}{2}$ and try question 3 again.

We do not agree.

$+\sqrt{25}$ is a positive square root, but it is a symbol representing the positive square root of 25, not -25 .

Please return to page $\frac{184}{2}$ and try question 2 again.

The symbol $\sqrt{}$ alone does not make the number irrational. If the radicand can be factored into two equal factors, then the expression is rational.

Can you write .09 as the product of two equal decimal numbers? Of course you can. Yes,

$$.09 = (.3)(.3)$$

Hence, $\sqrt{.09} = .3$

and $.3 = \frac{3}{10}$

We both agree that the fraction $\frac{3}{10}$ is a rational number.

Please return to page $\frac{192}{2}$ and try question 9 again.

180
1

Yes, your choice is correct.

To make sure, we multiply your choice for the square root by itself.

Thus,

$$(3x^8)\ (3x^8) = 9x^{16}$$

When a number is written as the product of 2 like factors, one of the factors is the square root of the number. A simple rule for finding the square root of letters raised to exponents is to divide the exponents by 2. Thus, for example.

$$\sqrt{x^8\ y^{10}} \quad \text{is} \quad x^4y^5$$

Please proceed to question 6 which follows.

180
2

Question 6

Apply the proper principle to simplify

$$\sqrt{\frac{25a^2}{49b^4}}$$

which is the correct result?

(A) $\frac{5a}{7b^2}$

(B) $\frac{5a}{7b}$

(C) $\frac{5a^2}{7b^4}$

(D) None of these.

Very good. You made the correct choice.

What does the symbol $\sqrt[3]{N}$ mean?

If a number N is written as the product of 3 equal factors, then one of these factors is called the cube root of the number and is written

$$\sqrt[3]{N}$$

Thus, $64 = 4 \cdot 4 \cdot 4$

and 4 is the cube root of 64, or symbolically

$$\sqrt[3]{64} = 4$$

Please proceed to question 8 which follows.

Question 8

If

$$2^5 = 32$$

choose the correct statement.

(A) $\sqrt{32} = 5$

(B) $\sqrt[2]{32} = 5$

(C) $\sqrt[5]{32} = 2$

(D) $\sqrt[5]{2} = 32$

182
1

Let us help you get started.

We have to find the square root of 7.25 .

We know it is between 2 and 3 .

Take for your first guess, the number 2 .

Test this guess as follows:

$$\frac{7.25}{2.00} = 3.6 \text{ (rounded off to the nearest tenth)}$$

For your second guess, you will take

$$\frac{1}{2}(2.0 + 3.6)$$

Please continue.

Please return to page 202/2 and try question 12 again.

182
2

We don't agree.

Did you look at the value in the table? It reads 5.385

Now re-read the problem.

Please return to page 191/2 and try question 11 again.

Yes, your choice is correct.

The symbol $\sqrt{N}$, where N is a positive number, means the positive square root of N.

Thus,

$$\sqrt{49} = +7$$

Please proceed to question 4 which follows.

Question 4

Apply the principle that

$$\sqrt{a^2 \cdot b^2} = a \cdot b$$

to find the square root of 1225 , given that 25 is a factor of 1225 .

(A) 5·49

(B) 25·49

(C) 25

(D) 5·7

184
1

We agree. The square root of 81 is + 9 or - 9 .

To find the square root of 81 we ask ourself this question: What number multiplied by itself equals 81 ? Since

$$(+9)\ (+9) = 81$$

and

$$(-9)\ (-9) = 81$$

there are two square roots.

The question asks for the negative square root; hence, -9 is the correct answer.

Please proceed to question 2 which follows.

184
2

Question 2

Choose the positive square root of - 25 .

(A) +5

(B) -5

(C) + $\sqrt{25}$

(D) None of these.

You have learned the rule:

$$\sqrt{\frac{p}{q}} = \frac{\sqrt{p}}{\sqrt{q}}$$

Thus,

$$\sqrt{\frac{1}{9}} = \frac{\sqrt{1}}{\sqrt{9}}$$

$$\sqrt{\frac{1}{9}} = \frac{1}{3}$$

Do you still think that you made the correct choice?

Please return to page 192/2 and try question 9 again.

.....

Let us refresh your memory on this procedure by doing a similar question together.

Problem: Find the square root of 18.4 to the nearest tenth.

Solution:

Our first guess is 4

We test this guess: $\frac{18.4}{4.0} = 4.6$

Our second guess is: $\frac{1}{2}(4.0 + 4.6) = 4.3$

We test this guess: $\frac{18.4}{4.3} = 4.3$ (to the nearest tenth)

Hence, the square root of 18.4 is 4.3 , correct to one place of decimals. Note that we stopped when the divisor was equal to the quotient.

Please return to page 197/2 and try question 10 again.

186
1

Very good. Your choice is correct.

The rule states that

$$\sqrt{\frac{p}{q}} = \frac{\sqrt{p}}{\sqrt{q}}$$

Please proceed to question 7 which follows.

Question 7

Choose the value of

$$\sqrt[3]{64}$$

(A) 4

(B) 8

(C) 2

(D) None of these.

186
2

It appears that you had every step correct except the last. In the final rounding off process, you find that you have to drop two digits. The proper procedure is to look at the first of these only. If it is less than 5 the previous digits are kept without any change. If the first figure being dropped is 5 or more, then we increase by 1 the last digit being kept. It is not proper to drop the last digit, round off, drop another digit and round off again and so on. As an example, 8.249 when rounded off to the nearest tenth, is 8.2 and not 8.3.

Please return to page 208/2 and try question 13 again.

If $2^5 = 32$

the number 32 is expressed as the product of five equal factors.

Thus, $32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

In general, if a number is expressed as the product of n equal factors, then one of these factors is the n^{th} root of the number. The symbol for n^{th} root is $\sqrt[n]{}$.

Please return to page 181/2 and try question 8 again.

.....

Since the description of this number shows it to be an infinite decimal with no uniform repeating cycle, this number is irrational.

Please return to page 200/2 and try question 17 again.

188
1

The number 5.3487 is a decimal that terminates; there are no more numbers after the number 7 .

A number expressible as a terminating decimal is a rational number. It can be expressed as a fraction.

$$\frac{53487}{10000}$$

Please return to page 199/2 and try question 15 again.

.....

188
2

We do not agree.

One of the letters does have the correct answer next to it.
Please examine the other choices more carefully.

Please return to page 205/2 and try question 18 again.

Very good. You made the correct choice.

The square of a positive number is a positive number. The square of a negative number is likewise positive.

Therefore, there is no number which we have met thus far whose square is -25. Is there a number whose square is negative?

Yes, $(5i)$, and it belongs to the set of imaginary numbers and you will not study such numbers until you take a second year of algebra.

Please proceed to question 3 which follows.

Question 3

Choose the correct value for $\sqrt{49}$

(A) $\pm$ 7

(B) + 7

(C) - 7

(D) $\pm \sqrt{7}$

190
1

Very good. You made the correct choice.

We know that one factor of 1225 is 25 . To find the other factor, we divide

$$\frac{1225}{25} = 49$$

Thus, $1225 = 25 \times 49$

We have a rule that states

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

Hence, $\sqrt{1225} = \sqrt{25 \times 49} = \sqrt{25} \cdot \sqrt{49}$

But, $\sqrt{25} = 5$ and $\sqrt{49} = 7$

Thus, the correct answer is

5×7

Please proceed to question 5 which follows.

190
2

Question 5

Apply your knowledge to simplify

$$\sqrt{9x^{16}}$$

(A) $3x^4$

(B) $\pm 3x^4$

(C) $3x^8$

(D) $\pm 3\sqrt{x^{16}}$

(E) None of these.

Very good. You made the correct choice.

As illustrated in the Introductory Note at the beginning of this segment, we proceed as follows:

To find the square root of 52.3 we take as our first guess, the number 7.

We test this guess: $\frac{52.3}{7.0} = 7.5$ (rounded off to the nearest tenth)

Our second guess is: $\frac{1}{2}(7.0 + 7.5) = 7.3$ (to the nearest tenth)

We test this guess: $\frac{52.3}{7.3} = 7.3$ (rounded off to the nearest tenth)

Hence, 3 is the correct answer to this question.

Please proceed to question 11 which follows.

.....

Question 11

Using the square root table on page 542 of your text, state which is the square root of 29 rounded off to the nearest tenth.

(A) 5.4

(B) 5.3

(C) 5.39

(D) None of these

192
1

Yes, your choice is correct.

When we write $2^5 = 32$

we mean that $32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

Thus, the number 32 is written as the product of 5 equal factors. One of these equal factors is, therefore, the fifth root of 32 . That is

$$\sqrt[5]{32} = 2$$

Please proceed to question 9 which follows.

.....

192
2

Question 9

Which of the following do you recognize as irrational?

(A) $\sqrt{4}$

(B) $\sqrt{.09}$

(C) $\sqrt{\frac{1}{9}}$

(D) $\sqrt{5}$

193/1

Squaring a value larger than 6 must give a value larger than 36 . The number .39 is certainly smaller than 36 . How did you place the decimal point in your answer?

Please return to page 208/2 and try question 13 again.

.....

193/2

This is like saying that a person was born in New York City, but was not born in the United States. The set of real numbers is the much larger set.
The set of rational numbers is a subset of the set of reals.

Hence, your choice cannot be correct.

Please return to page 200/2 and try question 17 again.

194
1

The expression

$$6.\overline{123}$$

is a repeating decimal. The bar over the numbers 23 signifies that these numbers keep repeating indefinitely. A number expressible as a repeating decimal is a rational number. It can be expressed as a fraction:

$$\begin{aligned} N &= 6.\overline{123} \\ 1000\,N &= 6123.\overline{23} \\ 10\,N &= 61.\overline{23} \\ \hline 990\,N &= 6062 \\ N &= \frac{6062}{990} = \frac{3031}{495} \end{aligned}$$

Please return to page 199/2 and try question 15 again.

194
2

We do not agree.

For example, the number $3\frac{1}{7}$ is a rational number.

This number is an approximation of the value of $\overline{77}$

The number $\overline{77}$ is irrational, but the approximation is a mixed number and is rational. However, this is not your only error. Please re-examine each choice.

Please return to page 207/2 and try question 18 again.

195/1

You seem to be having a little difficulty with putting the decimal point in the right place. Let us consider a similar question. Find the square root of 750 if it is given that $\sqrt{7.5} = 2.74$ (to the nearest hundredth).

Now, $750 = 100(7.5)$

and $\sqrt{750} = \sqrt{100(7.5)}$

Applying the rule that $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$

we can write $\sqrt{100(7.5)} = \sqrt{100} \cdot \sqrt{7.5}$

But, we know that $\sqrt{100} = 10$

hence $\sqrt{750} = 10(2.74) = 27.4$

Please return to page 203/2 and try question 14 again.

195/2

If you considered the meaning of the words rational and irrational from the standpoint of English only, you would have avoided this choice. Rational and irrational are opposites. Just as it is impossible for something to be true and not true at the same time, it is impossible for a number to be both rational and irrational.

Please examine the other choices more carefully before making a decision.

Please return to page 205/2 and try question 17 again.

196
1

Very good. You made the correct choice.

Clearly, the mixed number $3\frac{1}{7}$ is a rational number. Furthermore,

$$\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$$

and $\frac{2}{3}$ is a rational number. All the other choices consist of irrational numbers multiplied by a rational number. Such products are always irrational.

Please proceed to question 19 which follows.

196
2

Question 19

The square root of 15625 is a three digit number.

If you apply the method you have learned to find the value of the square root of 15625, which is the middle digit of the result?

(A) 0

(B) 1

(C) 2

(D) 3

Very good. You made the correct choice.

The number $\sqrt{5}$ is not a rational number. In order for the square root of an integer to be rational, it must be the square of an integer. Thus $\sqrt{9}$ is rational since $9 = 3^2$ and $\sqrt{64}$ is an integer since $64 = 8^2$.

Since $2^2 = 4$

and $3^2 = 9$

$= 2 < \sqrt{5} < 3$

Therefore, 5 is not the square of an integer; hence, it is irrational.

Please proceed to question 10 which follows.

.....

Question 10

The square root of 52.3 is larger than 7 .

Apply the method explained in the Introductory Note to calculate the tenths figure if $\sqrt{52.3}$ is rounded off to the nearest tenth.

(A) 1

(B) 2

(C) 3

(D) None of these.

198
1

We do not agree.
Take your answer, —.06 and square the absolute value; you should get .0036

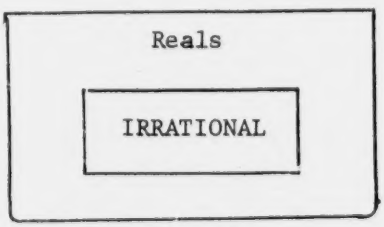
Always check out your choice for reasonableness.

Please return to page 208/2 and try question 13 again.

.....

198
2

The set of real numbers is the larger set.
The irrational numbers are a subset of the set of reals. Examine the sketch made below, and note that the set of irrational numbers is embedded in the set of reals.



Do you now see why your choice is incorrect?

Please return to page 200/2 and try question 17 again.

We agree.

If we look at the table on page 409 of the text, we find that $\sqrt{28}$ to the nearest hundredth is 5.29

Now, $2800 = (100)(28)$

and $\sqrt{2800} = \sqrt{(100)(28)}$

Using the rule that the square root of the product of two real numbers is equal to the product of their square roots, we get

$$\sqrt{(100)(28)} = \sqrt{100} \cdot \sqrt{28}$$

But, the square root of 100 is 10 . We, therefore, have

$$\sqrt{2800} = 10 \cdot \sqrt{28} = 10(5.29), \text{ and}$$
$$10(5.29) = 52.9$$

Since the question asks for both square roots, the answer is

$$\pm 52.9$$

Please proceed to question 15 which follows.

.....

Question 15

Which of the following do you recognize as an irrational number?

(A) $-\sqrt{.64}$

(B) 5.3487

(C) $6.1\overline{23}$

(D) None of these.

200
1

Yes, your choice is correct in as much as all the other choices are wrong.

Thus, $-\sqrt{64} = -8$

and -8 is a rational number;

5.3487

is a decimal which terminates and is, therefore, a rational number;

$6.1\overline{23}$

is a repeating decimal and repeating decimals represent rational numbers.

Please proceed to question 16 which follows.

200
2

Question 16

A number

$N = 1.01001000100001...$

is written by starting with the digit 1 followed by a decimal point, then a zero, then 1 , then 2 zeros, then 1 , then 3 zeros, then 1 , and continuing by inserting one more zero each time and then the digit 1 .

Choose which statement is true.

(A) N is real and rational

(B) N is real but irrational

(C) N is rational but not real

(D) N is irrational and not real.

201/1

You should know that the set of rational numbers is a proper subset of the real numbers. Furthermore, the set of irrational numbers is also a proper subset of the set of real numbers.

Are you now ready to make another choice?

Please return to page 205/2 and try question 17 again.

201/2

If you were asked to make a rough guess of the value of the square root of 15625 what would this guess be?

Let us try guessing together. We will guess the number 50 . The product of 50 and 50 is 2500 . This guess is, therefore, much too small. Let us try

$$(100)(100) = 10000$$

This guess is much better since both 15625 and 10000 are five digit numbers. Can we make a still closer guess? How about 120? Note that

$$(120)(120) = 14400$$

If necessary, re-read the procedure explained in the Introductory Note and continue with this question.

Please return to page 196/2 and try question 19 again.

202
1

Reading the value from the table, we get 5.385 .
Since we were asked to get the result to the nearest tenth, we drop the figures after the 3 . Since the first digit to be dropped is an 8 we increase the 3 . Therefore, you have made the correct choice.

Please proceed to question 12 which follows.

.....

202
2

Question 12

Apply your knowledge to calculate $\sqrt{7.25}$ correctly to the nearest tenth.

Which is the correct figure?

(A) 5
(B) 6
(C) 7
(D) 8

The illustration suggests that we should write .39 as the fraction $\frac{39}{100}$.

The $\sqrt{.39}$ becomes $\frac{\sqrt{39}}{\sqrt{100}}$

From the table $\sqrt{39} = 6.245$

Since $\sqrt{100} = 10$

we divide 6.245 by 10 getting .6245

Remember that division by 10 merely shifts the decimal point one place to the left. In order to round off .6245 to the nearest hundredth, we must drop the last two digits (45), the resulting number is somewhere between .62 and .63. Since the digits being dropped are less than half (45 out of 100), the original number is nearer .62 and we round off to that value.

Therefore, the value of $-\sqrt{.39}$

to the nearest hundredth is $-\sqrt{.62}$

Hence, you made the correct choice.

Please proceed to question 14 which follows.

Question 14

Since 2800 equals 100 times 28, apply your knowledge of the use the square root table to find the two square roots of 2800 to the nearest tenth.

(A) $\pm$ 5.29

(B) $\pm$ .503

(C) $\pm$ 52.9

(D) $\pm$ 529.0

204
1

If choice B stated that neither

$$\frac{\sqrt{2}}{2} \quad \text{nor} \quad 2\sqrt{2}$$

is rational, then it would have been the correct choice. Note that the $\sqrt{2}$ is irrational and multiplying or dividing an irrational number by a rational number yields an irrational number.

Please return to page 207/2 and try question 18 again.

204
2

Let us repeat the meaning of an irrational number.

A number is said to be irrational if it <u>cannot</u> be expressed either as a positive or negative whole number or as a positive or negative common fraction.

This is not the case with $-\sqrt{64}$.

Please return to page 199/2 and try question 15 again.

Very good. You made the correct choice.

This number is a non-terminating and non-repeating decimal.

This is in fact the definition of a number which is real and irrational.

Please proceed to question 17 which follows below.

Question 17

Choose which of the following combinations are possible as descriptions of a real number.

(A) Both rational and irrational

(B) Neither rational nor irrational

(C) Either rational or irrational

(D) None of these.

206
1

Very good. You made the correct choice.

Let us do this question together.

In order to find the square root of 7.25 we take as our first guess the number 2 since we know

$$2 < \sqrt{7.25} < 3$$

We test this guess: $\frac{7.25}{2.00} = 3.6$ (to the nearest tenth)

For our second guess, we take $\frac{1}{2}(2.0 + 3.6) = 2.8$

We test this guess: $\frac{7.25}{2.8} = 2.6$ (to the nearest tenth)

For our next guess, we average the new guess and the quotient and take

$$\frac{1}{2}(2.8 + 2.6) = 2.7$$

Testing this last guess, we get

$$\frac{7.25}{2.7} = 2.7$$ (rounded off to the nearest tenth)

If the two factors are equal, then we have found the square root. Hence 7 is the correct answer to this question.

Please proceed to question 13 which follows.

206
2

Question 13

Use the square root table in your text, and by following example 3a on page 409, apply the procedure illustrated to find to the nearest hundredth, the value of

$$-\sqrt{.39}$$

(A) -.62 (B) -.63

(C) -6.2 (D) -.06

We agree.

The set of real numbers contains all those numbers which are either rational or irrational. That is, the set of reals contains all the rational and irrational numbers.

Please proceed to question 18 which follows.

Question 18

Apply your knowledge and select the letter which labels the correct statement about the following numbers:

I. $3\frac{1}{7}$ II. $\frac{\sqrt{2}}{2}$ III. $\sqrt{\frac{4}{9}}$ IV. $\frac{\sqrt{6}}{2}$ V. $2\sqrt{2}$

(A) All the above numbers are irrational.

(B) Only $\frac{\sqrt{2}}{2}$ and $2\sqrt{2}$ are rational.

(C) Only $3\frac{1}{7}$ and $\sqrt{\frac{4}{9}}$ are rational.

(D) All the above numbers are rational.

$\frac{208}{1}$

Very good.

You made the correct choice.

We have to find the square root of 15625

For our first guess, we will take 120

We now test this guess: $\frac{15625}{120} = 130$ (to the nearest tenth)

For our second guess we take $\frac{1}{2}(120 + 130) = 125$

Testing this guess, we get $\frac{15625}{125} = 125$

Hence, the middle digit of the square root of 15625 is 2 .

You have now completed this Volume.

You should now complete the remaining problems from your HOMEWORK ASSIGNMENT in preparation for the TEST on VOLUME 15 .